Oct. 19

find $\frac{e}{m}$ CRT demo →
bring magnets & compass

Feedback: Separation of Variables will need more practice (not for Ch 5–7 needed → when we do review for final)

Read §5.1.3. (pages 216–222)

HW # 25 5.1, 5.2 a, b, 5.3

draw FBD

R, R−d, d, a

So far

q_1, q_2, q_3 … static

Q: $\vec{F} = Q\vec{E}$, $\vec{E}$-field

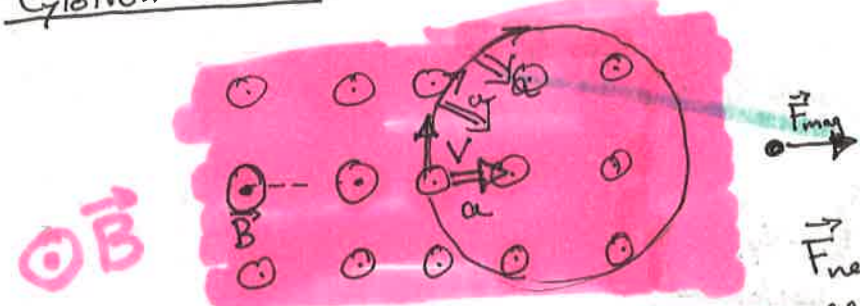
for Fr reading: steady currents cause $\vec{B}$-field ← §5.2

now

$$\boxed{\vec{F}_{mag} = Q\,\vec{v}\times\vec{B}}$$

Note: weird

Cylotron Motion



$\vec{F}_{net} = m\vec{a}$

use spherical coord.:

$\vec{F}_{mag} = m a_r(-\hat{r})$

Exercise:

uniform circular motion

← for problems 5.1 & 5.3

magnitude radially:

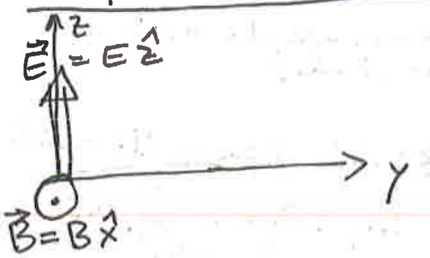
$$F_{mag} = m a$$

$$Q\,v\,B\,\underbrace{\sin\left(\tfrac{\pi}{2}\right)}_{=1} = m\,\frac{v^2}{R}$$

$$Q\,B\,R = m v = p \qquad \text{Eq. (5.3)}$$

(63)

Cycloid Motion



$$\vec{F}_{net} = Q(\vec{E} + \vec{v} \times \vec{B}) = m\vec{a}$$

$$QE\hat{z} + Q\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & \dot{y} & \dot{z} \\ B & 0 & 0 \end{vmatrix} = m(\ddot{y}\hat{y} + \ddot{z}\hat{z})$$

DEs for y and z:

$$+QB\dot{z} = m\ddot{y}$$

$$QE - QB\dot{y} = m\ddot{z}$$

$\xrightarrow{\omega = \frac{QB}{m}}$

$$\ddot{y} = \omega\dot{z}$$

$$\ddot{z} = \omega\left(\frac{E}{B} - \dot{y}\right)$$

Solve: Outline

step 1: rewrite in terms of v_y and v_z

I $\quad \dot{v}_y = \omega v_z$

II $\quad \dot{v}_z = \omega\left(\frac{E}{B} - v_y\right)$

step 2: take derivative of 1st & plug into second into II

$$\frac{dI}{dt} \quad \ddot{v}_y = \omega\dot{v}_z \rightarrow \dot{v}_z = \frac{1}{\omega}\ddot{v}_y$$

$$\frac{1}{\omega}\ddot{v}_y = \omega\left(\frac{E}{B} - v_y\right)$$

$$\ddot{v}_y = \omega^2\left(\frac{E}{B} - v_y\right)$$

step 3:

$$\ddot{v}_y + \omega^2 v_y = \overbrace{\omega^2\frac{E}{B}}^{\text{inhom.}}$$

$=0$ homog. equ. $\quad \ddot{v}_y = -\omega^2 v_y$

step 3.1: Solve homog. equ. $\quad v_y = A\cos(\omega t) + B(\sin(\omega t))$

step 3.2: Find inhomog. DE solution: $v_y = \text{const.} \rightarrow \ddot{v}_y + \omega^2 v_y = \omega^2\frac{E}{B}$, $\ddot{v}_y = 0$, $v_y = \frac{E}{B}$

step 3.3: $v_y = \text{homog.} + \text{inhom.}$
$= A\cos(\omega t) + B(\sin(\omega t)) + \frac{E}{B}$

Step 4: integrate $v_y(t)$

$$y(t) = \underbrace{\frac{A}{\omega}}_{C_2}\sin(\omega t) - \underbrace{\frac{B}{\omega}}_{C_1}\cos(\omega t) + \frac{E}{B}t + C_3$$

step 5 : $$v_z \overset{I}{=} \frac{1}{\omega}\overset{\ddot{y}}{\dot{v}_y} = \frac{1}{\omega}\left(-\omega^2 C_2 \sin(\omega t) - \omega^2 C_1 \cos(\omega t)\right)$$

$$= -\omega C_1 \cos(\omega t) - \omega C_2 \sin(\omega t)$$

integrate

$$z(t) = -C_1 \sin(\omega t) + C_2 \cos(\omega t) + C_4$$

$$\boxed{\begin{aligned} y(t) &= C_1 \cos(\omega t) + C_2 \sin(\omega t) + \frac{E}{B}t + C_3 \\ z(t) &= C_2 \cos(\omega t) - C_1 \sin(\omega t) + C_4 \end{aligned}}$$

← for pr. 5.2

C_1, C_2, C_3, C_4 via initial conditions

page 215 : $y(t=0) = z(t=0) = v_y(t=0) = v_z(t=0) = 0$

$y(t=0)=0 \quad C_1 + 0 + 0 + C_3 = 0 \rightarrow C_1 = -C_3 \quad C_3 = 0$

$z(t=0) = 0 \quad C_2 - 0 + C_4 = 0 \rightarrow C_2 = -C_4 \quad C_4 = +\frac{E}{B\omega}$

$\dot{y}(t=0) = 0 \quad 0 + C_2\omega + \frac{E}{B} + 0 = 0 \rightarrow C_2 = -\frac{E}{B\omega}$

$\dot{z}(t=0) = 0 \quad 0 - C_1\omega = 0 \rightarrow C_1 = 0$

$$\left.\begin{aligned} y(t) &= -\frac{E}{B\omega}\sin(\omega t) + \frac{E}{B}t \\ z(t) &= -\frac{E}{B\omega}\cos(\omega t) + \frac{E}{B\omega} \end{aligned}\right\}$$

rewrite $R = \frac{E}{\omega B}$

$$\boxed{(y - R\omega t)^2 + (z - R)^2 = R^2}$$

Cycloid

Phys 331 :
- shortest time roller coaster shape

and T indep. of amplitude
→ for clocks

Magnetic Forces

$$dW_{mag} = \vec{F}_{mag} \cdot d\vec{l} = Q(\vec{v} \times \vec{B}) \cdot \vec{v}\,dt$$

$$dW_{mag} = 0$$