

Equations

$$\mathbf{F}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r^2} \hat{\mathbf{n}} \quad \mathbf{F} = QE \quad \hat{\mathbf{n}} = \mathbf{r} - \mathbf{r}'$$

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{n}}_i \quad \mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{\mathbf{n}} \quad \oint_S \mathbf{E} \cdot d\mathbf{a} = \frac{1}{\epsilon_0} Q_{\text{enc}} \quad \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0} \rho$$

$$\mathbf{E} = \frac{\sigma}{2\epsilon_0} \hat{\mathbf{n}} \quad \nabla \times \mathbf{E} = 0 \quad V(\mathbf{r}) = - \int_{\mathcal{O}}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l} \quad \mathbf{E} = -\nabla V$$

$$\nabla^2 V = -\frac{\rho}{\epsilon_0} \quad V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i} \quad V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}')}{r} d\tau' \quad \mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \frac{\sigma}{\epsilon_0} \hat{\mathbf{n}}$$

$$W = \frac{1}{2} \int_{\mathcal{V}} \rho V d\tau = \frac{\epsilon_0}{2} \int_{\text{all space}} E^2 d\tau$$

$$d\tau = dx dy dz = s ds d\phi dz = r^2 \sin \theta dr d\theta d\phi$$

$$C \equiv Q/\Delta V = Q/(V_+ - V_-) \quad \sigma = -\epsilon_0 \left(\frac{\partial V^{\text{above}}}{\partial n} - \frac{\partial V^{\text{below}}}{\partial n} \right) \Big|_{\text{at the surface}}$$

$$V(r, \theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos \theta)$$

$$P_0(x) = 1 \quad P_1(x) = x \quad P_2(x) = \frac{1}{2}(3x^2 - 1) \quad P_3(x) = \frac{1}{2}(5x^3 - 3x)$$

$$\int_0^a \sin\left(\frac{n\pi y}{a}\right) \sin\left(\frac{n'\pi y}{a}\right) dy = \begin{cases} 0 & \text{for } n \neq n' \\ a/2 & \text{for } n = n' \end{cases}$$

$$\int_{-1}^1 P_l(x) P_{l'}(x) dx = \int_0^\pi P_l(\cos \theta) P_{l'}(\cos \theta) \sin \theta d\theta = \begin{cases} 0 & \text{for } l \neq l' \\ \frac{2}{(2l+1)} & \text{for } l = l' \end{cases}$$

$$V_{\text{mono}} = \frac{1}{4\pi\epsilon_0} \frac{1}{r} \int dq = \frac{1}{4\pi\epsilon_0} \frac{Q_{\text{tot}}}{r}$$

$$\mathbf{p} = \int \mathbf{r}' \rho(\mathbf{r}') d\tau' \quad \mathbf{p} = \sum_i q_i \mathbf{r}'_i$$

$$V_{\text{dip}} = \frac{1}{4\pi\epsilon_0} \frac{1}{r^2} \int r' \cos \alpha' dq' = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot \hat{\mathbf{r}}}{r^2}$$

$$\mathbf{E}_{\text{dip}} = \frac{p}{4\pi\epsilon_0 r^3} (2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\theta}) \quad \mathbf{E}_{\text{dip}} = \frac{1}{4\pi\epsilon_0 r^3} (3(\mathbf{p} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} - \mathbf{p})$$

$$\mathbf{F} = q\mathbf{E} \quad \mathbf{N} = \mathbf{p} \times \mathbf{E} \quad \mathbf{F} = (\mathbf{p} \cdot \nabla) \mathbf{E} \quad \sigma_b = (\mathbf{P} \cdot \hat{\mathbf{n}}) \Big|_{\text{at surface}} \quad \rho_b = -\nabla \cdot \mathbf{P}$$

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} \frac{\hat{\mathbf{n}} \cdot \mathbf{P}(\mathbf{r}')}{r'^2} d\tau' = \frac{1}{4\pi\epsilon_0} \left[\oint_S \frac{\sigma_b}{r} da' + \int_{\mathcal{V}} \frac{\rho_b}{r} d\tau' \right]$$

$$\rho = \rho_b + \rho_f \quad \mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \quad \nabla \cdot \mathbf{D} = \rho_f \quad \oint_S \mathbf{D} \cdot d\mathbf{a} = Q_{f,\text{enc}}$$

$$\text{linear dielectric:} \quad \mathbf{D} = \epsilon \mathbf{E} = \epsilon_0 \epsilon_r \mathbf{E} = \epsilon_0 (1 + \chi_e) \mathbf{E} \quad \mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$$

$$(E_{\text{above}}^{\perp} - E_{\text{below}}^{\perp}) \Big|_{\text{at surface}} = \frac{\sigma}{\epsilon_0} \quad (\mathbf{E}_{\text{above}}^{\parallel} - \mathbf{E}_{\text{below}}^{\parallel}) \Big|_{\text{at surface}} = \mathbf{0}$$

$$(D_{\text{above}}^{\perp} - D_{\text{below}}^{\perp}) \Big|_{\text{at surface}} = \sigma_f \quad (\mathbf{D}_{\text{above}}^{\parallel} - \mathbf{D}_{\text{below}}^{\parallel}) \Big|_{\text{at surface}} = (\mathbf{P}_{\text{above}}^{\parallel} - \mathbf{P}_{\text{below}}^{\parallel}) \Big|_{\text{at surface}}$$

$$V_{\text{above}} \Big|_{\text{at surface}} = V_{\text{below}} \Big|_{\text{at surface}} \quad \left(\frac{\partial V_{\text{above}}}{\partial n} - \frac{\partial V_{\text{below}}}{\partial n} \right) \Big|_{\text{at surface}} = -\frac{\sigma}{\epsilon_0}$$

$$\text{linear dielectric:} \quad \left(\epsilon_{\text{above}} \frac{\partial V_{\text{above}}}{\partial n} - \epsilon_{\text{below}} \frac{\partial V_{\text{below}}}{\partial n} \right) \Big|_{\text{at surface}} = -\sigma_f$$

$$\mathbf{I} = \frac{dq}{dt} \hat{\mathbf{l}} = I \hat{l}_{\parallel} = I \hat{v} = \lambda \mathbf{v} \quad \mathbf{K} = \frac{d\mathbf{I}}{dl_{\perp}} = \sigma \mathbf{v} \quad \mathbf{J} = \frac{d\mathbf{I}}{da_{\perp}} = \rho \mathbf{v}$$

$$\mathbf{F}_{\text{mag}} = q(\mathbf{v} \times \mathbf{B}) \quad \mathbf{F}_{\text{mag}} = \int I d\mathbf{l} \times \mathbf{B} \quad \mathbf{F}_{\text{mag}} = \int \mathbf{K} \times \mathbf{B} da \quad \mathbf{F}_{\text{mag}} = \int \mathbf{J} \times \mathbf{B} d\tau$$

$$\mathbf{B} = \frac{\mu_0}{4\pi} \int \frac{I d\mathbf{l}' \times \hat{\mathbf{n}}}{r'^2} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad \oint_{\mathcal{P}} \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} \quad \nabla \cdot \mathbf{B} = 0$$

$$W = \frac{1}{2} \int_{\text{all space}} \mathbf{D} \cdot \mathbf{E} d\tau$$

$$\mathbf{B} = \nabla \times \mathbf{A} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad \oint_{\mathcal{P}} \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} \quad \nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}')}{r} d\tau' \quad \mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{K}(\mathbf{r}')}{r} da' \quad \mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{I}(\mathbf{r}')}{r} dl'$$

$$B_{\text{above}}^{\perp} - B_{\text{below}}^{\perp} = 0 \quad \mathbf{B}_{\text{above}}^{\parallel} - \mathbf{B}_{\text{below}}^{\parallel} = \mu_0 \mathbf{K} \times \hat{\mathbf{n}}$$

$$\left(H_{\text{above}}^{\perp} - H_{\text{below}}^{\perp} \right) \Big|_{\text{at surface}} = - \left(M_{\text{above}}^{\perp} - M_{\text{below}}^{\perp} \right) \Big|_{\text{at surface}} \quad \left(\mathbf{H}_{\text{above}}^{\parallel} - \mathbf{H}_{\text{below}}^{\parallel} \right) \Big|_{\text{at surface}} = \mathbf{K}_f \times \hat{\mathbf{n}} \Big|_{\text{at surface}}$$

$$\mathbf{A}_{\text{above}} = \mathbf{A}_{\text{below}} \quad \frac{\partial \mathbf{A}_{\text{above}}}{\partial n} - \frac{\partial \mathbf{A}_{\text{below}}}{\partial n} = -\mu_0 \mathbf{K}$$

$$\mathbf{J}_b = \nabla \times \mathbf{M} \quad \mathbf{K}_b = (\mathbf{M} \times \hat{\mathbf{n}}) \Big|_{\text{at surface}}$$

$$\mathbf{m} = \int I d\mathbf{a} \quad \mathbf{B}_{\text{dip}} = \frac{\mu_0 m}{4\pi r^3} (2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\theta}) \quad \mathbf{B}_{\text{dip}} = \frac{\mu_0}{4\pi r^3} (3(\mathbf{m} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} - \mathbf{m})$$

$$\mathbf{N} = \mathbf{m} \times \mathbf{B} \quad \mathbf{F} = \nabla (\mathbf{m} \cdot \mathbf{B})$$

$$\oint_{\mathcal{P}} \mathbf{H} \cdot d\mathbf{l} = I_{f,\text{enc}} \quad \nabla \times \mathbf{H} = \mathbf{J}_f \quad \mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}$$

$$\text{linear magnetic material:} \quad \mathbf{M} = \chi_m \mathbf{H} \quad \mathbf{B} = \mu_0 (1 + \chi_m) \mathbf{H} = \mu_0 \mu_r \mathbf{H}$$

$$\mathcal{E} = IR \quad \mathbf{J} = \sigma \mathbf{E} \quad P = IV = I^2 R \quad \mathcal{E} = \oint \mathbf{f} \cdot d\mathbf{l} \quad \mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt} \int_{\mathcal{S}} \mathbf{B} \cdot d\mathbf{a}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \Phi_2 = MI_1 \quad \Phi = LI \quad W_{\text{mag}} = \frac{1}{2} LI^2 = \frac{1}{2\mu_0} \int_{\text{all space}} B^2 d\tau$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \mathbf{J} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

for Maxwell Equations see equation sheets from last pages of book

$$\oint_S \mathbf{D} \cdot d\mathbf{a} = Q_{f,\text{encl}} \quad \oint_S \mathbf{B} \cdot d\mathbf{a} = 0 \quad \oint_P \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{a} \quad \oint_P \mathbf{H} \cdot d\mathbf{l} = I_{f,\text{encl}} + \frac{d}{dt} \int_S \mathbf{D} \cdot d\mathbf{a}$$

$$(D_{\text{above}}^\perp - D_{\text{below}}^\perp) \Big|_{\text{at surface}} = \sigma_f \quad (B_{\text{above}}^\perp - B_{\text{below}}^\perp) \Big|_{\text{at surface}} = 0$$

$$(\mathbf{E}_{\text{above}}^\parallel - \mathbf{E}_{\text{below}}^\parallel) \Big|_{\text{at surface}} = \mathbf{0} \quad (\mathbf{H}_{\text{above}}^\parallel - \mathbf{H}_{\text{below}}^\parallel) \Big|_{\text{at surface}} = \mathbf{K}_f \times \hat{\mathbf{n}}$$

Useful Integrals and Trig Identities

$$\int \frac{x}{(a^2 + x^2)^{3/2}} dx = \frac{-1}{\sqrt{a^2 + x^2}} \quad \int \frac{1}{(a^2 + x^2)^{3/2}} dx = \frac{x}{a^2 \sqrt{a^2 + x^2}}$$

$$\int \frac{1}{\sqrt{a^2 + x^2}} dx = \ln(x + \sqrt{a^2 + x^2})$$

$$\int \sin^2(ax) dx = \frac{x}{2} - \frac{1}{4a} \sin(2ax) \quad \int x \sin^2(ax) dx = \frac{x^2}{4} - \frac{x}{4a} \sin(2ax) - \frac{1}{8a^2} \cos(2ax)$$

$$\cos(\alpha + \beta) = \cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta) \quad \sin(\alpha + \beta) = \sin(\alpha) \cos(\beta) + \cos(\alpha) \sin(\beta)$$

$$\sin^2\left(\frac{\theta}{2}\right) = \frac{1}{2}(1 - \cos(\theta)) \quad \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right) = \frac{1}{2} \sin(\theta)$$

$$\text{Law of cosines: } C^2 = A^2 + B^2 - 2AB \cos(\theta)$$