

Equations

$$\mathbf{F}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r^2} \hat{\mathbf{n}} \quad \mathbf{F} = QE \quad \hat{\mathbf{n}} = \mathbf{r} - \mathbf{r}'$$

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{n}}_i \quad \mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{\mathbf{n}} \quad \oint_S \mathbf{E} \cdot d\mathbf{a} = \frac{1}{\epsilon_0} Q_{\text{enc}} \quad \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0} \rho$$

$$\mathbf{E} = \frac{\sigma}{2\epsilon_0} \hat{\mathbf{n}} \quad \nabla \times \mathbf{E} = 0 \quad V(\mathbf{r}) = - \int_{\mathcal{O}}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l} \quad \mathbf{E} = -\nabla V$$

$$\nabla^2 V = -\frac{\rho}{\epsilon_0} \quad V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i} \quad V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}')}{r} d\tau' \quad \mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \frac{\sigma}{\epsilon_0} \hat{\mathbf{n}}$$

$$W = \frac{1}{2} \int_{\mathcal{V}} \rho V d\tau = \frac{\epsilon_0}{2} \int_{\text{all space}} E^2 d\tau$$

$$d\tau = dx dy dz = s ds d\phi dz = r^2 \sin \theta dr d\theta d\phi$$

$$C \equiv Q/\Delta V = Q/(V_+ - V_-) \quad \sigma = -\epsilon_0 \left(\frac{\partial V^{\text{above}}}{\partial n} - \frac{\partial V^{\text{below}}}{\partial n} \right) \Big|_{\text{at the surface}}$$

$$V(r, \theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos \theta)$$

$$P_0(x) = 1 \quad P_1(x) = x \quad P_2(x) = \frac{1}{2}(3x^2 - 1) \quad P_3(x) = \frac{1}{2}(5x^3 - 3x)$$

$$\int_0^a \sin\left(\frac{n\pi y}{a}\right) \sin\left(\frac{n'\pi y}{a}\right) dy = \begin{cases} 0 & \text{for } n \neq n' \\ a/2 & \text{for } n = n' \end{cases}$$

$$\int_{-1}^1 P_l(x) P_{l'}(x) dx = \int_0^\pi P_l(\cos \theta) P_{l'}(\cos \theta) \sin \theta d\theta = \begin{cases} 0 & \text{for } l \neq l' \\ \frac{2}{(2l+1)} & \text{for } l = l' \end{cases}$$

$$V_{\text{mono}} = \frac{1}{4\pi\epsilon_0} \frac{1}{r} \int dq = \frac{1}{4\pi\epsilon_0} \frac{Q_{\text{tot}}}{r}$$

$$\mathbf{p} = \int \mathbf{r}' \rho(\mathbf{r}') d\tau' \quad \mathbf{p} = \sum_i q_i \mathbf{r}'_i$$

$$V_{\text{dip}} = \frac{1}{4\pi\epsilon_0} \frac{1}{r^2} \int r' \cos \alpha' dq' = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot \hat{\mathbf{r}}}{r^2}$$

$$\mathbf{E}_{\text{dip}} = \frac{p}{4\pi\epsilon_0 r^3} (2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\theta}) \quad \mathbf{E}_{\text{dip}} = \frac{1}{4\pi\epsilon_0 r^3} (3(\mathbf{p} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} - \mathbf{p})$$

$$\mathbf{F} = q\mathbf{E} \quad \mathbf{N} = \mathbf{p} \times \mathbf{E} \quad \mathbf{F} = (\mathbf{p} \cdot \nabla) \mathbf{E} \quad \sigma_b = (\mathbf{P} \cdot \hat{\mathbf{n}}) \Big|_{\text{at surface}} \quad \rho_b = -\nabla \cdot \mathbf{P}$$

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} \frac{\hat{\mathbf{n}} \cdot \mathbf{P}(\mathbf{r}')}{r^2} d\tau' = \frac{1}{4\pi\epsilon_0} \left[\oint_S \frac{\sigma_b}{r} da' + \int_{\mathcal{V}} \frac{\rho_b}{r} d\tau' \right]$$

$$\rho = \rho_b + \rho_f \quad \mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \quad \nabla \cdot \mathbf{D} = \rho_f \quad \oint_S \mathbf{D} \cdot d\mathbf{a} = Q_{f,\text{enc}}$$

$$\text{linear dielectric:} \quad \mathbf{D} = \epsilon \mathbf{E} = \epsilon_0 \epsilon_r \mathbf{E} = \epsilon_0 (1 + \chi_e) \mathbf{E} \quad \mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$$

$$(E_{\text{above}}^{\perp} - E_{\text{below}}^{\perp}) \Big|_{\text{at surface}} = \frac{\sigma}{\epsilon_0} \quad (\mathbf{E}_{\text{above}}^{\parallel} - \mathbf{E}_{\text{below}}^{\parallel}) \Big|_{\text{at surface}} = \mathbf{0}$$

$$(D_{\text{above}}^{\perp} - D_{\text{below}}^{\perp}) \Big|_{\text{at surface}} = \sigma_f \quad (\mathbf{D}_{\text{above}}^{\parallel} - \mathbf{D}_{\text{below}}^{\parallel}) \Big|_{\text{at surface}} = (\mathbf{P}_{\text{above}}^{\parallel} - \mathbf{P}_{\text{below}}^{\parallel}) \Big|_{\text{at surface}}$$

$$V_{\text{above}} \Big|_{\text{at surface}} = V_{\text{below}} \Big|_{\text{at surface}} \quad \left(\frac{\partial V_{\text{above}}}{\partial n} - \frac{\partial V_{\text{below}}}{\partial n} \right) \Big|_{\text{at surface}} = -\frac{\sigma}{\epsilon_0}$$

$$\text{linear dielectric:} \quad \left(\epsilon_{\text{above}} \frac{\partial V_{\text{above}}}{\partial n} - \epsilon_{\text{below}} \frac{\partial V_{\text{below}}}{\partial n} \right) \Big|_{\text{at surface}} = -\sigma_f$$