Physics 333 **Electromagnetic Theory I** **Fall 2022**

Equations

$$\mathbf{F}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{Qq}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}} \qquad \mathbf{F} = Q\mathbf{E} \qquad \boldsymbol{\mathscr{r}} = \mathbf{r} - \mathbf{r}'$$

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{\mathscr{r}_i^2}\hat{\boldsymbol{\mathscr{r}}}_i \qquad \mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\mathrm{d}q}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}} \qquad \oint_{\mathcal{S}} \mathbf{E}\cdot d\mathbf{a} = \frac{1}{\epsilon_0}Q_{\text{enc}} \qquad \nabla\cdot\mathbf{E} = \frac{1}{\epsilon_0}\rho$$

$$\mathbf{E} = \frac{\sigma}{2\epsilon_0}\,\hat{\mathbf{n}} \qquad \nabla\times\mathbf{E} = 0 \qquad V(\mathbf{r}) = -\int_{\mathcal{O}}^{\mathbf{r}} \mathbf{E}\cdot d\mathbf{l} \qquad \mathbf{E} = -\nabla V$$

$$\nabla^2 V = -\frac{\rho}{\epsilon_0} \qquad V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{\mathscr{r}_i} \qquad V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\mathbf{r}')}{\mathscr{r}}\,d\tau' \qquad \mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \frac{\sigma}{\epsilon_0}\,\hat{\mathbf{n}}$$

$$W = \frac{1}{2}\int_{\mathcal{V}} \rho V\,d\tau = \frac{\epsilon_0}{2}\int_{\text{all space}} E^2\,d\tau$$

$$d\tau = dx\,dy\,dz = s\,ds\,d\phi\,dz = r^2\sin\theta\,dr\,d\theta\,d\phi$$

$$C \equiv Q/\Delta V = Q/\left(V_+ - V_-\right) \qquad \sigma = -\epsilon_0\left(\frac{\partial V^{\text{above}}}{\partial n} - \frac{\partial V^{\text{below}}}{\partial n}\right)\Bigg|_{\text{at the surface}}$$

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right) P_l(\cos\theta)$$

$$P_0(x) = 1 \qquad P_1(x) = x \qquad P_2(x) = \frac{1}{2}(3x^2 - 1) \qquad P_3(x) = \frac{1}{2}(5x^3 - 3x)$$

$$\int_0^a \sin\left(\frac{n\pi y}{a}\right)\sin\left(\frac{n'\pi y}{a}\right)\mathrm{d}y = \begin{cases} 0 & \text{for } n \neq n' \\ a/2 & \text{for } n = n' \end{cases}$$

$$\int_{-1}^{1} P_l(x)P_{l'}(x)\mathrm{d}x = \int_0^{\pi} P_l(\cos\theta)P_{l'}(\cos\theta)\sin\theta\,\mathrm{d}\theta = \begin{cases} 0 & \text{for } l \neq l' \\ \frac{2}{(2l+1)} & \text{for } l = l' \end{cases}$$

$$V_{\text{mono}} = \frac{1}{4\pi\epsilon_0}\frac{1}{r}\int \mathrm{d}q = \frac{1}{4\pi\epsilon_0}\frac{Q_{\text{tot}}}{r}$$

$$\mathbf{p} = \int \mathbf{r}'\rho(\mathbf{r}')\,d\tau' \qquad \mathbf{p} = \sum_i q_i\mathbf{r}'_i$$

$$V_{\text{dip}} = \frac{1}{4\pi\epsilon_0}\frac{1}{r^2}\int r'\cos\alpha'\mathrm{d}q' = \frac{1}{4\pi\epsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2}$$

$$\mathbf{E}_{\text{dip}} = \frac{p}{4\pi\epsilon_0 r^3}(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\theta}) \qquad \mathbf{E}_{\text{dip}} = \frac{1}{4\pi\epsilon_0 r^3}\Big(3(\mathbf{p}\cdot\hat{\mathbf{r}})\,\hat{\mathbf{r}} - \mathbf{p}\Big)$$

$$\mathbf{F} = q\mathbf{E} \qquad \mathbf{N} = \mathbf{p}\times\mathbf{E} \qquad \mathbf{F} = (\mathbf{p}\cdot\nabla)\,\mathbf{E} \qquad \sigma_b = (\mathbf{P}\cdot\hat{\mathbf{n}})\Big|_{\text{at surface}} \qquad \rho_b = -\nabla\cdot\mathbf{P}$$

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}}\frac{\hat{\boldsymbol{\mathscr{r}}}\cdot\mathbf{P}(\mathbf{r}')}{\mathscr{r}^2}d\tau' = \frac{1}{4\pi\epsilon_0}\left[\oint_{\mathcal{S}}\frac{\sigma_b}{\mathscr{r}}da' + \int_{\mathcal{V}}\frac{\rho_b}{\mathscr{r}}d\tau'\right]$$

$$\rho = \rho_b + \rho_{\text{f}} \qquad \mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P} \qquad \nabla\cdot\mathbf{D} = \rho_{\text{f}} \qquad \oint_{\mathcal{S}}\mathbf{D}\cdot d\mathbf{a} = Q_{\text{f,enc}}$$

linear dielectric: $\mathbf{D} = \epsilon\mathbf{E} = \epsilon_0\epsilon_r\mathbf{E} = \epsilon_0(1+\chi_e)\mathbf{E}$ $\qquad \mathbf{P} = \epsilon_0\chi_e\mathbf{E}$

$$\left(E^{\perp}_{\text{above}} - E^{\perp}_{\text{below}}\right)\Big|_{\text{at surface}} = \frac{\sigma}{\epsilon_0} \qquad \left(\mathbf{E}^{\parallel}_{\text{above}} - \mathbf{E}^{\parallel}_{\text{below}}\right)\Big|_{\text{at surface}} = \mathbf{0}$$

$$\left(D^{\perp}_{\text{above}} - D^{\perp}_{\text{below}}\right)\Big|_{\text{at surface}} = \sigma_{\text{f}} \qquad \left(\mathbf{D}^{\parallel}_{\text{above}} - \mathbf{D}^{\parallel}_{\text{below}}\right)\Big|_{\text{at surface}} = \left(\mathbf{P}^{\parallel}_{\text{above}} - \mathbf{P}^{\parallel}_{\text{below}}\right)\Big|_{\text{at surface}}$$

$$V_{\text{above}}\Big|_{\text{at surface}} = V_{\text{below}}\Big|_{\text{at surface}} \qquad \left(\frac{\partial V_{\text{above}}}{\partial n} - \frac{\partial V_{\text{below}}}{\partial n}\right)\Big|_{\text{at surface}} = -\frac{\sigma}{\epsilon_0}$$

linear dielectric: $\left(\epsilon_{\text{above}}\frac{\partial V_{\text{above}}}{\partial n} - \epsilon_{\text{below}}\frac{\partial V_{\text{below}}}{\partial n}\right)\Big|_{\text{at surface}} = -\sigma_{\text{f}}$

$$\mathbf{I} = \frac{\mathrm{d}q}{\mathrm{d}t}\hat{I} = I\hat{l}_{\parallel} = I\hat{v} = \lambda\mathbf{v} \qquad \mathbf{K} = \frac{\mathrm{d}\mathbf{I}}{\mathrm{d}l_{\perp}} = \sigma\mathbf{v} \qquad \mathbf{J} = \frac{\mathrm{d}\mathbf{I}}{\mathrm{d}a_{\perp}} = \rho\mathbf{v}$$

$$\mathbf{F}_{\text{mag}} = q(\mathbf{v}\times\mathbf{B}) \qquad \mathbf{F}_{\text{mag}} = \int I\,\mathrm{d}\mathbf{l}\times\mathbf{B} \qquad \mathbf{F}_{\text{mag}} = \int \mathbf{K}\times\mathbf{B}\,\mathrm{d}a \qquad \mathbf{F}_{\text{mag}} = \int \mathbf{J}\times\mathbf{B}\,\mathrm{d}\tau$$

$$\mathbf{B} = \frac{\mu_0}{4\pi}\int\frac{I\,d\mathbf{l}'\times\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2} \qquad \nabla\times\mathbf{B} = \mu_0\mathbf{J} \qquad \oint_{\mathcal{P}}\mathbf{B}\cdot d\mathbf{l} = \mu_0 I_{\text{enc}} \qquad \nabla\cdot\mathbf{B} = 0$$

$$W = \frac{1}{2} \int\limits_{\text{all space}} \mathbf{D} \cdot \mathbf{E} \, \mathrm{d}\tau$$

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \qquad \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} \qquad \nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}')}{\mathscr{r}} d\tau' \qquad \mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{K}(\mathbf{r}')}{\mathscr{r}} da' \qquad \mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{I}(\mathbf{r}')}{\mathscr{r}} dl'$$

$$B^{\perp}_{\text{above}} - B^{\perp}_{\text{below}} = 0 \qquad \mathbf{B}^{\parallel}_{\text{above}} - \mathbf{B}^{\parallel}_{\text{below}} = \mu_0 \mathbf{K} \times \hat{\mathbf{n}}$$

$$H^{\perp}_{\text{above}} - H^{\perp}_{\text{below}} = -\left(M^{\perp}_{\text{above}} - M^{\perp}_{\text{below}}\right) \qquad \mathbf{H}^{\parallel}_{\text{above}} - \mathbf{H}^{\parallel}_{\text{below}} = \mathbf{K}_{\text{f}} \times \hat{\mathbf{n}}$$

$$\mathbf{A}_{\text{above}} = \mathbf{A}_{\text{below}} \qquad \frac{\partial \mathbf{A}_{\text{above}}}{\partial n} - \frac{\partial \mathbf{A}_{\text{below}}}{\partial n} = -\mu_0 \mathbf{K}$$

$$\mathbf{J}_b = \nabla \times \mathbf{M} \qquad \mathbf{K}_b = (\mathbf{M} \times \hat{\mathbf{n}}) \Big|_{\text{at surface}}$$

$$\mathbf{m} = \int I \mathrm{d}\mathbf{a} \qquad \mathbf{B}_{\text{dip}} = \frac{\mu_0 m}{4\pi} \frac{1}{r^3} (2 \cos\theta \, \hat{\mathbf{r}} + \sin\theta \, \hat{\theta}) \qquad \mathbf{B}_{\text{dip}} = \frac{\mu_0}{4\pi} \frac{1}{r^3} \Big(3(\mathbf{m} \cdot \hat{\mathbf{r}}) \, \hat{\mathbf{r}} - \mathbf{m} \Big)$$

$$\mathbf{N} = \mathbf{m} \times \mathbf{B} \qquad \mathbf{F} = \nabla \left(\mathbf{m} \cdot \mathbf{B} \right)$$

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{\text{f,enc}} \qquad \nabla \times \mathbf{H} = \mathbf{J}_{\text{f}} \qquad \mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}$$

Useful Integrals and Trig Identities

$$\int \frac{x}{\left(a^2+x^2\right)^{3/2}}\ \mathrm{d}x = \frac{-1}{\sqrt{a^2+x^2}} \qquad \int \frac{1}{\left(a^2+x^2\right)^{3/2}}\ \mathrm{d}x = \frac{x}{a^2\sqrt{a^2+x^2}}$$

$$\int \frac{1}{\sqrt{a^2+x^2}}\ \mathrm{d}x = \ln\left(x+\sqrt{a^2+x^2}\right)$$

$$\int \sin^2(a\,x)\ \mathrm{d}x = \frac{x}{2} - \frac{1}{4a}\sin(2a\,x) \qquad \int x\ \sin^2(a\,x)\ \mathrm{d}x = \frac{x^2}{4} - \frac{x}{4a}\sin(2a\,x) - \frac{1}{8a^2}\cos(2a\,x)$$

$$\cos\left(\alpha+\beta\right) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) \qquad \sin\left(\alpha+\beta\right) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\sin^2\left(\frac{\theta}{2}\right) = \frac{1}{2}\left(1-\cos(\theta)\right) \qquad \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) = \frac{1}{2}\sin(\theta)$$

Law of cosines: $C^2 = A^2 + B^2 - 2AB\cos(\theta)$