

Homework Assignment #6

(due Sept. 7, 2022, at beginning of class)

1. Griffiths 1.45
2. Griffiths 1.48
3. In homework #2.3 and #2.4 you determined the divergence and the curl for

- $\mathbf{v}_a = y^2 \hat{\mathbf{x}} + (2xy + z^2) \hat{\mathbf{y}} + 2yz \hat{\mathbf{z}}$
- $\mathbf{v}_b = -xy \hat{\mathbf{x}} + 2yz \hat{\mathbf{y}} + 7xz \hat{\mathbf{z}}$
- $\mathbf{v}_c = x^2 \hat{\mathbf{x}} + 3xz^2 \hat{\mathbf{y}} - 2xz \hat{\mathbf{z}}$

- (i) Which of these vectors can be expressed as $\mathbf{v} = \nabla T$? For the corresponding \mathbf{v} find T .
- (ii) Which of these vectors can be expressed as $\nabla \times \mathbf{A}$? For the corresponding \mathbf{v} find \mathbf{A} .

Hint: Choose one of the \mathbf{A} -components to be zero, for example $A_y = 0$. To not give away, which of the vectors above you will use, let me explain the next step with some other vector $\mathbf{v}_d = 3xy^2 \hat{\mathbf{x}} + (y^2 - y^3) \hat{\mathbf{y}} + 2yz \hat{\mathbf{z}}$. In this case choosing $A_y = 0$ means $v_{d,x} = (\nabla \times \mathbf{A})_x = \left(\frac{\partial}{\partial y} A_z - \frac{\partial}{\partial z} A_y \right) = \frac{\partial}{\partial y} A_z = 3xy^2$ and therefore $A_z = xy^3 + f(x, z)$, where $f(x, z)$ indicates a function which may depend on x and/or z , but not on y . Similarly you use $v_{d,y}$ and $v_{d,z}$.