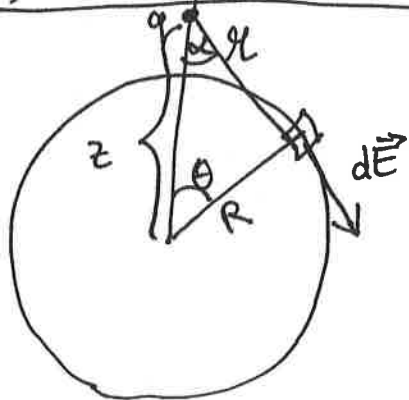


Homework Assignment #13
(due Sep 26, 2022, at the beginning of class)

1. Griffiths 3.1
2. Griffiths 3.3
3. Griffiths 3.4

Hints for HW 13.3 Griffiths 3.4



by symmetry

$$\vec{E}_{ave} = E_{ave,z} \hat{z} = -E_{ave} \hat{z} \quad \left(\frac{q}{4\pi\epsilon_0 r^2} \right)$$

$$\text{so } E_{ave,z} = -\frac{1}{4\pi R^2} \oint_{\text{sphere surface}} dE_z da = -\frac{1}{4\pi R^2} \int dE \cos\alpha da$$

$R^2 \sin\theta d\theta d\phi$

So we need to express all variables in terms of θ and constants R and z :

$$\int_0^{2\pi} d\phi = 2\pi$$

$$r^2 = R^2 + z^2 - 2Rz \cos\theta$$

for $\cos\alpha$ we use $R^2 = z^2 + r^2 - 2rz \cos\alpha \rightarrow$ solve for $\cos\alpha$

put into $E_{ave,z}$ above
... & simplify

$$E_{ave,z} = -\frac{q}{16\pi^2 R^2 \epsilon_0} \int_0^\pi \frac{z - R \cos\theta}{(R^2 + z^2 - 2Rz \cos\theta)^{3/2}} R^2 \sin\theta d\theta d\phi$$

substitution $\tilde{u} = \cos\theta \quad \frac{d\tilde{u}}{d\theta} = -\sin\theta$

$$E_{ave,z} = -\dots \int_{-1}^1 \frac{(z - R\tilde{u})}{(R^2 + z^2 - 2Rz\tilde{u})^{3/2}} R^2 \cancel{\sin\theta} \frac{d\tilde{u}}{(-\cancel{\sin\theta})}$$

use integration by parts

use $u = (z - R\tilde{u}) \quad v' = (R^2 + z^2 - 2Rz\tilde{u})^{-3/2}$

then distinguish $z > R$ and $z < R$ for taking $\sqrt{\dots}$ to get positive value