Physics 333 **Electromagnetic Theory I** **Fall 2022**

Homework Assignment #31

(due Nov 28, 2022, at the beginning of class)

1. Griffiths 6.15

   Hints: Follow the hints provided in the problem statement, including to use Eq. (3.65). The hint to use Eq. (6.24) corresponds to

$$\begin{aligned} H^{\perp}_{\text{above}} - H^{\perp}_{\text{below}} &= -(M^{\perp}_{\text{above}} - M^{\perp}_{\text{below}}) \\ H^{\perp}_{\text{outside}} - H^{\perp}_{\text{inside}} &= (M^{\perp}_{\text{inside}} - M^{\perp}_{\text{outside}}) \\ H^{\perp}_{\text{outside}} - H^{\perp}_{\text{inside}} &= M^{\perp}_{\text{inside}} \\ -\frac{\partial W_{\text{outside}}}{\partial r}\bigg|_{r=R} + \frac{\partial W_{\text{inside}}}{\partial r}\bigg|_{r=R} &= M^{\perp}_{\text{inside}} = M\,\hat{\mathbf{z}}\cdot\,\hat{\mathbf{r}} = M\cos(\theta) \end{aligned}$$

   where the left side of the last line followed from $\mathbf{H} = -\vec{\nabla} W$ in spherical coordinates.

   To get a second boundary condition we use that

$$W(\mathbf{b}) - W(\mathbf{a}) = \int_{\mathbf{a}}^{\mathbf{b}} \vec{\nabla} W \cdot \mathrm{d}\vec{l} = -\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{H}\cdot \mathrm{d}\vec{l}$$

   from which follows for $|\mathbf{a}-\mathbf{b}| \to 0$ that $W(\mathbf{b}) - W(\mathbf{a}) \to 0$, therefore

$$W_{\text{inside}}(r,\theta) = W_{\text{outside}}(r,\theta)$$

   All this tells you that you may use the boundary conditions

$$\begin{aligned} (i) \qquad & W_{\text{inside}}(r,\theta) = W_{\text{outside}}(r,\theta) \\ (ii) \qquad & -\frac{\partial W_{\text{outside}}}{\partial r}\bigg|_{r=R} + \frac{\partial W_{\text{inside}}}{\partial r}\bigg|_{r=R} = M^{\perp}_{\text{inside}} = M\,\hat{\mathbf{z}}\cdot\,\hat{\mathbf{r}} = M\cos(\theta) \end{aligned}$$

2. Griffiths 6.16