Summary for Test 1

Vector Calculus

- draw vector field
- $\vec{A} \times \vec{B}$
- $\vec{\nabla} T$, $\vec{\nabla} \cdot \vec{v}$, $\vec{\nabla} \times \vec{v}$, $\nabla^2 T$
- $\int_{P} \vec{v} \cdot d\vec{l}$
- $\int_{S} \vec{v} \cdot d\vec{a}$
- $\int_{\mathcal{V}} T \, d\tau$
- fundamental theorems (check)
- δ - fct.
- potential

SUMMARY FOR TEST 2

δ-function

Electrostatics:

Coulomb's law $\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{\mathscr{r}^2} \hat{\mathscr{r}}$ $\vec{F} = q\vec{E}$

Electric Field $\vec{E} = \frac{1}{4\pi\epsilon_0} \sum \frac{q_i}{\mathscr{r}_i^2} \hat{\mathscr{r}}_i$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{\mathscr{r}^2} \hat{\mathscr{r}}$$

Gauss's law $\oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}$ $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

& applications

$$V(\vec{r}) = -\int_{\infty}^{\vec{r}} \vec{E} \cdot d\vec{\ell}'$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{\mathscr{r}}$$ and $\vec{E} = -\vec{\nabla} V$

$$W = \frac{1}{2} \int_{\mathcal{V}} \rho(r) V(r) = \frac{\epsilon_0}{2} \int_{\text{all space}} E^2 d\tau$$

- - - - -

next test: conductors & $C = \frac{Q}{\Delta V}$

SUMMARY FOR TEST 3

Conductors:

- $V = \text{const}$, $\vec{E} = 0$ inside, ...
- apply $\oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\varepsilon_0}$
- $C = \frac{Q}{\Delta V} = \frac{Q}{V_+ - V_-}$

Uniqueness Thms: apply (not proof)
↓
show that solution satisfies boundary conditions (b.c.)

Method of Images

Sketches: Fig. 1, Fig. 2 (image(s))
& Show that b.c. satisfied

$V = \ldots$

$\vec{F}_{on q}$

$$\sigma = -\varepsilon_0 \left(\frac{\partial V^{above}}{\partial n} - \frac{\partial V^{below}}{\partial n} \right) \Bigg|_{\text{at surface}}$$

$$q_{induced} = \int \sigma \, da$$

Separation of Variables:

sketch

$V = X(x)\, Y(y)\, Z(z)$ $\quad R(r)\, \Phi(\phi)\, \Theta(\theta)$ ...

$\nabla^2 V = 0$ & separate variables

solve DE (see Eq. sheet for what is provided)

Apply b.c. one by one (last via direct comparison or Fourier's Trick)

plug back into $V = \ldots$

$\sigma = \ldots$

$\vec{F} = q\vec{E} = -q \vec{\nabla} V$

Summary for Test 4

- Multipole Expansion:
  - * determine $\vec{p}$, V_{mono}, V_{dip}
  - * NOT: derivation of $\vec{E}_{dip}$ (HW 18.1)

- Determine * $\vec{E}_{dip}$ (general & special case) ($\vec{E}$ due to dipole)
  - * $\vec{N}$, $\vec{F}$ on dipole (& $\vec{F}$ on charge)

  & combinations

- σ_b, ρ_b & Gauss's Law

- Dielectric:
  - * determine & draw $\vec{D}$, $\vec{P}$, $\vec{E}$

    for general & for linear dielectric

    includes using Gauss's Law for $\vec{D}$ & $\vec{E}$

  - * $C = \frac{Q}{\Delta V}$ $\quad \Delta V = -\int_{-}^{+} \vec{E} \cdot d\vec{l}$

- NOT: Boundary Conditions & Separation of Variables

Summary for Test 5

- $\vec{D}, \vec{E}$ via Gauss's law then $W = \frac{1}{2}\int \vec{D}\cdot\vec{E}\, d\tau$
- Separation of Variables using bound. cond. for V and $\epsilon \frac{\partial V}{\partial n}$
- $\vec{F}_{mag} = q\vec{v}\times\vec{B}$ simpler case & cycloids
- $\vec{I}, \vec{K}, \vec{J}$ and corresponding $\vec{F}_{mag}$
- Bio Savart $\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\, d\vec{l}\times \hat{\mathscr{r}}}{\mathscr{r}^2}$ etc. for $\vec{K}, \vec{J}$

  & $\vec{F}_{mag}$
- Ampère's Law $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$

NOT: $\vec{A}$, bound. cond. for $\vec{A}, \vec{B}$, multipole expansion (incl. magnetic dipole)

Summary For Test 6

(Celebration of:) Ampère's Law $\int \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$ (needs sketch with Amperian Loop)

given $\vec{J}\ (\vec{J}_b, \vec{J}_f)$, $\vec{K}\ (\vec{K}_b, \vec{K}_f)$, $I\ (I_f)$

$\rightarrow \vec{B}, \vec{H}$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,')\,d\tau'}{\mathscr{r}} \qquad \text{etc.}$$

$$\vec{B} = \vec{\nabla}\times\vec{A} \qquad \vec{J} = \frac{1}{\mu_0}(\vec{\nabla}\times\vec{B})$$

$$\vec{m} = \int I\, d\vec{a}$$

$$\vec{B}_{dip}(\vec{r}) = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$$

$$\& \ \vec{B}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi r^3}\left[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}\right]$$

$$\vec{N} = \vec{m}\times\vec{B} \quad \text{and} \quad \vec{F} = \vec{\nabla}(\vec{m}\cdot\vec{B})$$

$$\vec{J}_b = \vec{\nabla}\times\vec{M} \quad \text{and} \quad \vec{K}_b = \vec{M}\times\hat{n}$$

(Ampère's Law & Interpretation)

$$\oint \vec{H}\cdot d\vec{l} = I_{f,encl} \quad \text{and} \quad \vec{B} = \mu_0(\vec{H}+\vec{M})$$

boundary conditions: derive & sketches of $\vec{M}, \vec{B}, \vec{H}$

Electrostatics & Magnetostatics

Summary for Content after Test 6

- Determine $\vec{B}$:

$$\vec{M} \longrightarrow \vec{J}_b, \vec{K}_b \xrightarrow[\text{Ampère's Law}]{} \vec{B}$$

$$\vec{M} \longrightarrow \vec{H} = -\vec{\nabla} W \xrightarrow[\text{separ. of variables}]{} W \rightarrow \vec{H} \rightarrow \vec{B}$$

$$I_f \xrightarrow[\text{Ampère's Law}]{} \vec{H} \rightarrow \vec{B}$$

- Boundary Conditions $\xrightarrow{\text{apply}}$ sketch for bar magnet $\vec{M}, \vec{B}, \vec{H}$

- $\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{a}$ $\quad$ $\mathcal{E} = IR$ and from before: $\Delta V = -\int_{-}^{+} \vec{E} \cdot d\vec{l}$

$$C = \frac{Q}{\Delta V}$$

- in matter $\vec{J} = \sigma(\vec{E} + \underbrace{\vec{v} \times \vec{B}}_{\text{neglected in examples}})$

- $\Phi = MI$

$\Phi = LI$

- $W = \frac{1}{2\mu}\int_{\text{all space}} B^2 d\tau = \frac{1}{2\mu_0}\left[\int_{\mathcal{V}} B^2 d\tau + \int_{S} (\vec{A} \times \vec{B}) \cdot d\vec{a}\right]$

$$W = \frac{1}{2} L I^2$$

- Maxwell Equations
  - correction
  - interpretation
  - differential $\longleftrightarrow$ integral
  - derive boundary conditions

Thank you for great semester with you!