In-Class Work: Mini-Project II

Today you will work in groups of two or three on assigned mini-projects (see below to which group and project you belong). You will work today more with the driven damped pendulum and will work with your group on further analysis of the driven damped pendulum, including the topic of chaos. You will work from 9:30-10:10 on your analysis, from 10:10-10:30 on your slides (for most of your projects two slides) and 10:30-10:52 each group will present for four minutes their results to the class.

The equation of motion for the driven damped pendulum is

$$\frac{d^2\theta}{dt^2} = \tilde{A}\,\cos\left(\tilde{\omega}_\mathrm{D}\,t\right) - \sin(\theta) - \tilde{\gamma}\,\frac{d\theta}{dt} \tag{4}$$

Mini-Project II.1 (Alex, Connor, and Luke)

Your group will investigate how the trajectories $\theta(t)$, $\omega(t)$, and $E(t)$ depend on the driving amplitude $\tilde{A}$. Your group will present first and the following groups will build on your results. You will therefore use the same parameters as the following groups. This implies that you need to use a time step Δt that is a equal to the driving period T_d devided by an integer number. That is, as discussed in last class

$$\Delta t = \frac{2\pi}{\tilde{\omega}_\mathrm{D} N_\mathrm{dt}}$$

You will use for the integer variable $N_\mathrm{dt} = 200$.

II 1a. Copy the program

```
~kvollmay/share.dir/inclass.dir/md7_miniII1_start.py
```

into your working directory. Familiarize yourself with the program and determine which value each parameter ($\tilde{A}$, ω_D, $\tilde{\gamma}$) is assigned in the program and also which initial conditions are used.

II 1b. Make graphs of $\theta(t), \omega(t)$ and $E(t)$ for the following values of $\tilde{A}$:

$$\tilde{A} = 0.95, 1.049, 1.053, 1.054, 1.07$$

Keep all other values in the program the same. For each of the above cases look at the transient and also enlarge the oscillations at the end of the simulation (or you may want to only print the last few thousand MD steps). You should notice different scenarios depending on $\tilde{A}$.

II 1c. Your group will need more than two slides. So your group should work on the slides not only after 30 min, but start right away with making clear xmgrace figures. For your first slide state clearly what your group's main goal is. For the following slides state which parameters initial conditions you chose. Indicate the transient on your xmgrace graph(s) as illustrated in last class and clearly label your axes and for each figure label the value of $\tilde{A}$. Make several slides for the different scenarios corresponding to the above $\tilde{A}$.

II 1d. Finish all slides for your talk and plan the words for your 4 min talk, and plan who of you two will say what. Put your talk slides in your `~/share.dir` and give read permission.

II 1e. (if time) In case your group gets all slides made and has time left, then redo your figures for the different initial condition $\theta_0 = -2.5$ and $\omega_0 = 0.0$.

II1e Put your slide and program in your `~/share.dir/` and give me read-permission.

Mini-Project II.2 (Katie and CJ)

II 2a. Copy the program

```
~kvollmay/share.dir/inclass.dir/md8_miniII2_start.py
```

into your working directory. Familiarize yourself with the program and determine which value each parameter ($\tilde{A}$, ω_D, $\tilde{\gamma}$) is assigned in the program and also which initial conditions are used.

Note that the program includes compared to last class also that angles are periodic, that is θ gets mapped to

$$-\pi < \theta \leq \pi$$

We need this mapping for all following to be able to check whether the pendulum motion is periodic. Goal of your project is investigate the so called phase space plot, which is $\omega(\theta)$, for a variety of $\tilde{A}$ values.

II 2b. The phase space plot, is a plot of ω versus θ. Any point in this phase space specifies a set of initial conditions, i.e. in principle the determined trajectory. So, it is the space of the degrees of freedom. Make graphs of $\omega(\theta)$ for the following values of $\tilde{A}$:

$$\tilde{A} = 0.95, 1.049, 1.053, 1.054, 1.07$$

Keep all other values in the program the same. For the first two cases of $\tilde{A}$ look at the transient . Think how to use xmgrace and the `out8*` data to get $\omega(\theta)$. Note that the given program allows you to either include the transient with `nmeasstart=0` or to cut out the transient and instead to look at the late times, that is to use `nmeasstart=nmax-100*nDeltat` . You should look at the resulting late time trajectories for all $\tilde{A}$ given above. You should notice different scenarios depending on $\tilde{A}$.

II 2c. Your group will need more than two slides. So your group should work on the slides not only after 30 min, but start right away with making clear xmgrace figures. For your first slide state clearly what your group's main goal is. Comment also on the period angle as explained above. For your following slides state clearly which parameters and initial conditions you chose. Indicate the transient on your xmgrace graph as illustrated in last class and clearly label your axes and for each figure label the value of $\tilde{A}$. Make several slides for the different scenarios corresponding to the above $\tilde{A}$.

II 2d. Finish all slides for your talk and plan the words for your 4 min talk, and plan who of you two will say what. Put your talk slides in your `~/share.dir` and give read permission.

Mini-Project II.3 (Houtan, Jonathan, and Ziqi)

II 3a. Copy the program

```
~kvollmay/share.dir/inclass.dir/md8_miniII3_start.py
```

into your working directory. Familiarize yourself with the program and determine which value each parameter ($\tilde{A}$, $\omega_{\rm D}$, $\tilde{\gamma}$) is assigned in the program and also which initial conditions are used.

Note that the program includes compared to last class also that angles are periodic, that is θ gets mapped to

$$-\pi < \theta \leq \pi$$

We need this mapping for all following to be able to check whether the pendulum motion is periodic. Goal of your project is investigate the so called Poincaré-plot (defined below) for a variety of $\tilde{A}$ values.

II 3b. To get an idea of the Poincaré plot you first should look at the phase space plot, which is a plot of ω versus θ. Any point in this phase space specifies a set of initial conditions, i.e. in principle the determined trajectory. So, it is the space of the degrees of freedom. Make graphs of $\omega(\theta)$ for $\tilde{A} = 0.95$ and for $\tilde{A} = 1.049$. You should notice that for the late time (which is what your program plots) that your phase space plot shows periodic behavior but different periodicity depending on $\tilde{A}$.

II 3c. Notice that for the above scenarios you would want to find out whether the trajectory repeats with $T_{\rm d}$ or with multiples of $T_{\rm d}$ or whether the trajectory never repeats. You can check this with the so called Poincaré plot, which is also a plot of $\omega(\theta)$ but you print instead only with the period $T_{\rm d}$. (Think how you can do this with the program. Please get me if this step seems a lot of work, because it should be only one value which you adjust.) Look at the resulting Poincaré plots for the above $\tilde{A}$ values. Make Poincaré plots for the following values of $\tilde{A}$:

$$\tilde{A} = 0.95, 1.049, 1.053, 1.054, 1.07$$

II 3d. For your first slide state clearly which parameters and initial conditions you chose and what your group's main goal is. The previous group will have shown the phase space plots, so make one slide to define how you get the Poincaré plot. Then show the different Poincaré plots for the above $\tilde{A}$ values. Clearly label your axes and for each figure label the value of $\tilde{A}$.

II 3e. Finish all slides for your talk and plan the words for your 4 min talk, and plan who of you two will say what. Put your talk slides in your `~/share.dir` and give read permission.

Mini-Project II.4 (Rajasri and Narayan)

II 4a. Copy the programs

```
~kvollmay/share.dir/inclass.dir/md8_miniII4_start.py
~kvollmay/share.dir/inclass.dir/md9_miniII4_start.py
```

into your working directory. To familiarize yourself with the programs and determine which value each parameter ($\omega_{\rm D}$, $\tilde{\gamma}$) is assigned in the program and also which initial conditions are used.

Note that the program includes compared to last class also that angles are periodic, that is θ gets mapped to

$$-\pi < \theta \leq \pi$$

We need this mapping for all following to be able to check whether the pendulum motion is periodic. Goal of your project is to investigate the so called bifurcation diagram. Your project presentation will summarize the results of the previous groups.

To help you understand the programs first look at

```
~kvollmay/share.dir/inclass.dir/md8_miniII4_start.py
```

Run it and look at the resulting data file with

```
xmgrace -block out8_A1.049.dat -bxy 2:3
```

The program is a plot of $\omega(\theta)$ for late times and prints only in time with period $T_{\rm d}$. It is a so called Poincaré plot.

II 4b. Now you are ready to look at the program for your group's task: the bifurcation diagram, that is a plot of $\theta(\tilde{A})$. Look very carefully at

```
~kvollmay/share.dir/inclass.dir/md9_miniII4_start.py
```

Read very carefully this program, what exactly it does. Then run the program (this program takes a while to finish the simulation run) and look at the output file. Make a figure of the resulting bifurcation diagram.

II 4c. For your first slide state clearly which parameters and initial conditions you chose and what your group's main goal is. The previous groups will have shown the phase space plots, and the Poincaré plots for various $\tilde{A}$ values. Make a slide to clearly define the bifurcation diagram. Then make a slide with your bifurcation diagram. Clearly label your axes and all parameters and initial conditions.

II 4d. (if time) You may also make a bifurcation diagram of not only $\theta(\tilde{A})$ but also of $\omega(\tilde{A})$ and $E(\tilde{A})$ with

```
xmgrace -block out9_bifurc_th0omega19.dat -bxy 1:2 -bxy 1:3 -b 1:4
```

II 4e. Finish all slides for your talk and plan the words for your 4 min talk, and plan who of you two will say what. Put your talk slides in your `~/share.dir` and give read permission.