In-Class Work: Mini-Project II

Today you will work in groups of two or three on assigned mini-projects (see below to which group and project you belong). You will work today more with the driven damped pendulum and will work with your group on further analysis of the driven damped pendulum, including the topic of chaos. You will work from 9:30-10:10 on your analysis, from 10:10-10:30 on your slides (for most of your projects two slides) and 10:30-10:52 each group will present for four minutes their results to the class.

The equation of motion for the driven damped pendulum is

$$\frac{d^2\theta}{dt^2} = \tilde{A}\,\cos\left(\tilde{\omega}_{\mathrm{D}}\,t\right) - \sin(\theta) - \tilde{\gamma}\,\frac{d\theta}{dt} \tag{5}$$

Remember, that in last class we derived this equation for the unitless quantities $\tilde{\theta}, \tilde{t}, \tilde{A}, \tilde{\omega}_{\mathrm{D}}$, and $\tilde{\gamma}$. Only for the sake of programming simplicity in the equation we changed notation, by using θ (instead of $\tilde{\theta}$) and t (instead of $\tilde{t}$), so the above equation is ready for implementation in the program (same what you used at the end of last class for the in-class MD task 6.).

Mini-Project II.1 (Andrew, Coby, Grant)

Your group will investigate how the trajectories $\theta(t)$, $\omega(t)$, and $E(t)$ depend on the driving amplitude $\tilde{A}$. Your group will present first and the following group will build on your results. You will therefore use the same parameters as the following group. This implies that you need to use a time step Δt that is a value equal to the driving period T_{d} devided by an integer number. That is, instead of choosing $\Delta t = 0.01$, you use

$$\Delta t = \frac{2\pi}{\tilde{\omega}_{\mathrm{D}} N_{\mathrm{dt}}}$$

You will use for the integer variable $N_{\mathrm{dt}} = 200$.

II 1a. Copy the program

```
~kvollmay/share.dir/inclass2019.dir/md7_miniII1_start.py
```

into your working directory. Familiarize yourself with the program and determine which value each parameter ($\tilde{A}$, ω_{D}, $\tilde{\gamma}$) is assigned in the program and also which initial conditions are used.

II 1b. Make graphs of $\theta(t), \omega(t)$ and $E(t)$ for the following values of $\tilde{A}$:

$$\tilde{A} = 0.95, 1.049, 1.053, 1.054, 1.07$$

Keep all other values in the program the same. For each of the above cases look at the transient and also enlarge the oscillations at the end of the simulation (or you may want to only print the last few thousand MD steps). You should notice different scenarios depending on $\tilde{A}$.

Helpful xmgrace-command:

```
xmgrace -block out7_A0.95.dat -bxy 1:2 -bxy 1:3 -bxy 1:4  -world 0 -4 100 4.0
```

and similarly for other files and other time windows. You can put labels on the graphs: go on the top right to "Window" then choose "Drawing objects" and then select "Text props" to increase the "size" to at least 150, and then choose "Text" and click on the figure on the place where you want text. In the string you can type any label and choose the appropriate color. To obtain E, so in italic font you type `\f{1}E` and to get θ you type `\xq` and for omega `\xw`. "Edit object" allows you to edit an already existing label. To get decent x-axis tick marks: click on top on `Plot` and then `Properties` and then adjust the Major spacing, which specifies for which tick marks there is a label. To compare the maxima and minima heights at late times you may want to add some horizontal line(s). You can do this also with "Window", then "Drawing Objects" and then "Line props" and " Line".

II 1c. Your group will need more than two slides. So your group should work on the slides not only after 30 min, but start right away with making clear xmgrace figures. For your first slide state clearly what your group's main goal is. For the following slides state which parameters initial conditions you chose. Indicate the transient on your xmgrace graph(s) as illustrated in last class and clearly label your axes and for each figure label the value of $\tilde{A}$. Make several slides for the different scenarios corresponding to the above $\tilde{A}$.

II 1d. Finish all slides for your talk and plan the words for your 4 min talk, and plan who of you three will say what. Put your talk slides in your `~/share.dir` and give read permission.

II1e. Put your slide and program in your `~/share.dir/` and give me read-permission.

II 1f. (if time) In case your group gets all slides made and has time left, then have a look at Mini-Project II.2.

Mini-Project II.2 (if time)

II 2a. Copy the program

```
~kvollmay/share.dir/inclass2019.dir/md8_miniII2_start.py
```

into your working directory. Familiarize yourself with the program and determine which value each parameter ($\tilde{A}$, ω_{D}, $\tilde{\gamma}$) is assigned in the program and also which initial conditions are used.

Note that the program includes compared to last class also that angles are periodic, that is θ gets mapped to

$$-\pi < \theta \leq \pi$$

We need this mapping for all following to be able to check whether the pendulum motion is periodic. Goal of this project is to investigate the so called phase space plot, which is $\omega(\theta)$, for a variety of $\tilde{A}$ values.

II 2b. The phase space plot, is a plot of ω versus θ. Any point in this phase space specifies a set of initial conditions, i.e. in principle the determined trajectory. So, it is the space of the degrees of freedom. Make graphs of $\omega(\theta)$ for the following values of $\tilde{A}$:

$$\tilde{A} = 0.95, 1.049, 1.053, 1.054, 1.07$$

Keep all other values in the program the same. For the first two cases of $\tilde{A}$ look at the transient . Think how to use xmgrace and the `out8*` data to get $\omega(\theta)$. Note that the given program allows you to either include the transient with `nmeasstart=0` or to cut out the transient and instead to look at the late times, that is to use `nmeasstart=nmax-100*nDeltat` . You should look at the resulting late time trajectories for all $\tilde{A}$ given above. You should notice different scenarios depending on $\tilde{A}$.

Mini-Project II.3 (Jeanine and JJ)

II 3a. Copy the program

```
~kvollmay/share.dir/inclass2019.dir/md8_miniII3_start.py
```

into your working directory. Familiarize yourself with the program and determine which value each parameter ($\tilde{A}$, $\omega_{\rm D}$, $\tilde{\gamma}$) is assigned in the program and also which initial conditions are used.

Note that the program includes compared to last class also that angles are periodic, that is θ gets mapped to

$$-\pi < \theta \leq \pi$$

We need this mapping for all following to be able to check whether the pendulum motion is periodic. Goal of your project is to investigate the so called Poincaré-plot (defined below) for a variety of $\tilde{A}$ values.

II 3b. To get an idea of the Poincaré plot you first should look at the phase space plot, which is a plot of ω versus θ. Any point in this phase space specifies a set of initial conditions, i.e. in principle the determined trajectory. So, it is the space of the degrees of freedom. Make graphs of $\omega(\theta)$ for $\tilde{A} = 0.95$ and for $\tilde{A} = 1.049$. You should notice that for the late time (which is what your program plots) that your phase space plot shows periodic behavior but different periodicity depending on $\tilde{A}$.

You might find the following xmgrace command useful

`xmgrace -block out8_A1.049.dat -bxy 2:3`

For further xmgrace tips, you may look at the miniprojectII 1b xmgrace commands. You can also label x-axis (and y-axis) by choosing on the top "Plot" and therein "Axis Properties" and the x-label you type in for "Axis Label: Label string:". Also here you can use to get θ in xmgrace `\xq` and for ω in xmgrace `\xw`. To get rid of the lines, go to "Plot" and therein to "Set Appearance" and for the Line Properties choose "None" and for the "Symbol Properties" choose for example "Circle".

II 3c. Notice that for the above scenarios you would want to find out whether the trajectory repeats with $T_{\rm d}$ or with multiples of $T_{\rm d}$ or whether the trajectory never repeats. You can check this with the so called Poincaré plot, which is also a plot of $\omega(\theta)$ but you print instead only with the period $T_{\rm d}$. (Think how you can do this with the program. Please get me if this step seems a lot of work, because it should be only one value which you adjust.) Look at the resulting Poincaré plots for the above $\tilde{A}$ values. Make Poincaré plots for the following values of $\tilde{A}$:

$$\tilde{A} = 0.95, 1.049, 1.053, 1.054, 1.07$$

II 3d. For your first slide state clearly which parameters and initial conditions you chose and what your group's main goal is. The other group will probably not have looked at phase space

plots, so show a few phase space plots and define then how you get the Poincaré plot. Then show the different Poincaré plots for the above $\tilde{A}$ values. Clearly label your axes and for each figure label the value of $\tilde{A}$.
Assuming that you print the Poincaré plot data for example into `out8_A1.049.dat` you might find the following xmgrace command helpful

```
xmgrace -block out8_A1.049.dat -bxy 2:3 -block out8_Poincare_A1.049.dat -bxy 2:3
```

II 3e. Finish all slides for your talk and plan the words for your 4 min talk, and plan who of you two will say what. Put your talk slides in your `~/share.dir` and give read permission.

II 3f. (If time) Have a look at the following mini-project II.4, which would be for getting the bifurcation diagram.

Mini-Project II.4 (If time)

II 4a. Copy the programs

```
~kvollmay/share.dir/inclass.dir/md9_miniII4_start.py
```

into your working directory. To familiarize yourself with the program confirm that it is for the same values of $(\omega_{\rm D},\ \tilde{\gamma})$ as you used in Mini-Project II.3.
Note that also here the program includes compared to last class also that angles are periodic, that is θ gets mapped to

$$-\pi < \theta \leq \pi$$

We need this mapping for all following to be able to check whether the pendulum motion is periodic. Goal of your project is to investigate the so called bifurcation diagram. Your project presentation will summarize the results of the previous groups.

II 4b. Now you are ready to look at the program for the bifurcation diagram, that is a plot of $\theta(\tilde{A})$ where θ is printed periodically printed with $T_{\rm D}$, so the same θ as in the Poincaré plot. Look very carefully at

```
~kvollmay/share.dir/inclass.dir/md9_miniII4_start.py
```

Read very carefully this program, what exactly it does. Then run the program (this program takes a while to finish the simulation run) and look at the output file. Make a figure of the resulting bifurcation diagram.

II 4c. For your first slide state clearly which parameters and initial conditions you chose and what your group's main goal is. The previous groups will have shown the phase space plots, and the Poincaré plots for various $\tilde{A}$ values. Make a slide to clearly define the bifurcation diagram. Then make a slide with your bifurcation diagram. Clearly label your axes and all parameters and initial conditions.

II 4d. (if time) You may also make a bifurcation diagram of not only $\theta(\tilde{A})$ but also of $\omega(\tilde{A})$ and $E(\tilde{A})$ with

```
xmgrace -block out9_bifurc_th0omega19.dat -bxy 1:2 -bxy 1:3 -bxy 1:4
```