# Driven Damped Pendulum

$$\frac{d^2\theta}{dt^2} = \tilde{A}\cos(\tilde{\omega}_D t) - \sin(\theta) - \tilde{\gamma}\frac{d\theta}{dt}$$

$\theta_0 = 0 \quad \omega_0 = 1.9 \quad \tilde{\omega}_D = \frac{2}{3} \quad \tilde{\gamma} = 0.5$

$\tilde{T}_D = \frac{2\pi}{\tilde{\omega}_D}$

| | $\tilde{A}=0.95$ | $\tilde{A}=1.049$ | $\tilde{A}=1.053$ | $\tilde{A}=1.054$ | $\tilde{A}=1.07$ |
|---|---|---|---|---|---|
| $\phi(t)$ (similarly $\omega(t)=\frac{d\theta}{dt}$) | $\tilde{T}_D$, transient | $2\tilde{T}_D$ | $4\tilde{T}_D$ | $8\tilde{T}_D$, periodic | never repeats |
| phase space plot / state space plot $\omega(\phi)$ | transient | | | 8 times around before repeat | infinite number of loops; never repeats |
| $\omega(\phi, t)$ trajectory | $T_D$ | | | | |
| Poincaré Plot | $\omega_{PC}$ | | | | infinite number of dots, fractal pattern |
| | period-one | period-two | period-four | period-eight | chaos |

Summary:

Bifurcation Diagram