In-Class Work: DLA — Fractal Dimension

We have been implementing the Diffusion Limited Aggregation (DLA) model by Witten and Sander [T.A. Witten, L.M. Sander, Phys. Rev. Lett **47**, 1400 (1981)]

8. Finished DLA program

You find the finished DLA program in `~kvollmay/share.dir/inclass2023.dir/classfractal8.py`

9. Fractal Dimension of DLA Cluster

9a. In last class we talked about the fractal dimension $d_{\rm f}$. Please use your notes for the meaning of b (was n in specific examples of Cantor set and Triadic Kochcurve) and N (which is the number of occupied sites). I will start today's class with guiding us through the solution for 9c.

9b. Now lets get ready to analyze the pattern of the DLA model. You will determine the fractal dimension of one pattern using the method of checking squares of length `b`, as just described in class.

To avoid having to run the DLA program again and again, let us first prepare one pattern, which you then will analyze in 9c. Run the program
`~kvollmay/share.dir/inclass2023.dir/classfractal8.py`
This program makes the file `bigDLAcluster.dat` (and a nice pdf-file `frame8.pdf` just for fun). (Or if you have your own finished DLA program, have a look at the last few lines of `classfractal8.py` to see how to write the file `bigDLAcluster.dat`.) Ensure that you run the program for `LATSIZE=500` and for `NPARTMAX=3000`. This will take a while, but we have to do this only once, because for the analysis we use `bigDLAcluster.dat`.

9c. Now you need a program which reads in the 224x224 matrix from your file `bigDLAcluster.dat`. You may use for this task
`~kvollmay/share.dir/inclass2023.dir/classfractal9start.py`
To get the fractal dimension $d_{\rm f}$ we use the following relation.

$$\ln(N) = d_{\rm f} * \ln(b) \tag{2}$$

where N is the number of occupied sites, c is some constant and b is the length of your square for which you count the number of occupied sites. You see that Eq.(2) defines d_f and it tells us that if we plot $\ln(N)$ as a function of $\ln(b)$ then we should get a line with slope $d_{\rm f}$. So our task is to get N and b. Add to your program that you count the number of occupied sites N for a lattice of lenght b, where you center your lattice of lenght b around the midpoint of your 224 x 224 lattice. Loop over the length of your lattice and print out $\ln(N)$ as a function of $\ln(b)$. Let's say you do
`classfractal9c.py > lnNoflnb.dat`
Hint: $\ln(N)$ is in python `np.log(N)`
You find an implementation of this task in `~kvollmay/share.dir/inclass2023.dir/classfractal9c.py`

9d. Next we fit a line (we can fit a line through the origin) to our data from 9c stored in file `lnNoflnb.dat`. For this we use gnuplot. So type in the command line "`gnuplot`". Then type

"`plot "lnNoflnb.dat"`". Define a function $f(x)$ by typing "`f(x) = a*x `". Now fit your data within the xrange [2.0,4.0] to a line by typing "`fit [2.0:4.0] f(x) "lnNoflnb.dat" via a`". The resulting a is the fractal dimension $d_{\rm f}$. You can look at the data and fit with "`plot "lnNoflnb.dat",f(x)`"
Play some with the fit range. Compare your fractal dimension with the expected value of 1.71

To make figure:

To save your gnuplot session in the file `DLAdf.gnu`, you type within gnuplot `save "DLAdf.gnu"`
When you start a new gnuplot session, you can load your gnuplot-session within gnuplot with `load "DLAdf.gnu"`
You get for example the xlabel with `set xlabel "ln(b)"` and to save your figure into a postscript file use within gnuplot for example `set term postscript landscape; set out "DLAdf.ps;replot`
To convert the DLAdf.ps into a pdf-file, you can use outside of gnuplot in the terminal window `ps2pdf DLAdf.ps`

10. History of DLA Particles (just for fun!)

To understand the shape of the DLA clusters, let us now indicate the history of the particles with their color: Print "5" for the first 500 particles, "6" for the next 500 particles, "7" for the following 500 particles etc.. Look at the final cluster picture (pdf-file).
Hint: If you have used your own program, instead of classfractal8.py, you might need to adjust your check if the random walker is next to a particle of the cluster.

11. The rest of the class you may use for working on the mini-project.