

# QBism and Greenberger-Horne-Zeilinger experiments: A simple worked example

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## Abstract

QBism is an interpretation of quantum mechanics developed to resolve many of the paradoxes that arise in more traditional interpretations taught to students, such as the Copenhagen interpretation. I provide a worked example of a simple thought experiment, using undergraduate-level quantum mechanics, that is articulated in the in language of, and informed by, the perspective of QBism. The thought experiment uses an entangled three-particle Greenberger-Horne-Zeilinger state.

## I. INTRODUCTION

Almost a century after the formulation of the modern theory of quantum mechanics, scholarly debate continues regarding the foundational underpinnings of the theory. The first two decades of the 21<sup>st</sup> century have seen the development of a new interpretation of the theory, called QBism. (In its earlier incarnations this interpretation was known as Quantum Bayesianism, but QBism has grown to encompass more than the term Bayesianism alone might imply.) In an article in this journal proponents of QBism make the claim that the interpretation “removes the paradoxes, conundra, and pseudo-problems that have plagued quantum foundations for the past nine decades.”<sup>1</sup> Other are not so convinced; in the pages of this journal QBism has been referred to as “a radical minority view among physicists” that isn’t really necessary to resolve foundational issues.<sup>2</sup>

Typical users of quantum mechanics do not need to be well-read in the quantum foundations literature to be successful scientists; the shut-up-and-calculate approach to quantum mechanics suffices in most professional applications. On the other hand, most users of quantum mechanics are at least aware of the paradoxes and unresolved issues that are addressed by the scholarship of quantum foundations, and many are puzzled by them, and interested in understanding at least the basics of the various approaches to resolving the issues. Understanding how the QBist perspective fits within the variety of approaches to quantum foundations is complicated for physicists outside of the field by the fact that QBism is couched in language that is unfamiliar to the typical physicist, and it is rooted in a subjective view of probability<sup>3-7</sup> that isn’t generally taught to students of physics. To help non-specialists gain insight into QBism, I demonstrate how the QBist perspective manifests itself in a simple worked example using undergraduate-level quantum mechanics. The example is a thought experiment using a three-particle version of the Einstein-Podolsky-Rosen (EPR) experiment,<sup>8</sup> that was constructed by Greenberger, Horne, and Zeilinger<sup>9-11</sup> (hereafter GHZ); the example demonstrates the incompatibility of quantum mechanics with the premises of the EPR program. The material in this paper provides a quantitative complement to the more qualitative discussion of a GHZ experiment in Chapter 14 of reference 12. (The discussion in this paper is narrowly focussed on the worked example, and does not provide a complete introduction to QBism, nor a critical review of the perspective.)

## II. SUMMARY OF QBISM

This section gives a very brief summary of QBism, highlighting the features of the perspective that will arise in the context of the worked example in the following sections. The reader interested in a more thorough introduction to QBism can turn to the material in von Baeyer’s 2016 book intended for a general audience,<sup>12</sup> Mermin’s 2012 Commentary in *Physics Today*<sup>13</sup> (with responses<sup>14</sup>), and the previously referenced article from this journal by Fuchs, Mermin, and Schack<sup>1</sup> (along with a response<sup>15</sup>). A comprehensive review article summarized the state of the field in 2013,<sup>16</sup> and a review from a philosophical perspective appeared in 2017;<sup>17</sup> other discussions outlining progress in the QBist program that go well beyond issues considered in this paper are also available.<sup>18,19</sup>

QBism is an interpretation of quantum mechanics that is informed by the perspectives of quantum information theory. To a QBist, a wavefunction (or state vector) does not represent an element of physical reality; rather, it is a construct of an individual *agent* who is a user of the quantum formalism. The state vector is a mathematical articulation of the individual agent’s degree of belief about the system, and it encapsulates the information that she can use to make probabilistic predictions of her future experiences, such as the outcome of an experiment she performs. “The QBist wavefunction . . . is not a universally agreed-upon, observer-independent formula, but an expression personal to each agent. It depends on each agent’s knowledge, and is thus subjective.”<sup>20</sup>

The agents discussed in this paper are assumed to be sophisticated *users* of the quantum mechanical formalism. State vectors are *assigned* by individual agents based on personal experience; that experience may include results of past experiments, education, and communication with other scientists. An agent will update her state vector based on new personal experiences, and calculate updated probabilities. To a QBist, subjective probabilities are expressed in a willingness to gamble on outcomes with appropriately determined odds. From this point of view, all state vectors are local, in the sense that every agent assigns a state vector based on the experience of that agent. Experiences of an agent provide information which the agent can use to update her state vector; there is no need to turn to concepts like ‘wavefunction collapse.’ Results of experiments do not reveal elements of reality (in the sense of EPR) — they simply provide information.

Statements like these can sound downright heretical to those of us educated in the tradi-

tion of objective science in which the scientist’s role is to distance herself from the features of the real world that are being interrogated. When applied in the context of a simple thought experiment these attitudes appear less distant than they might initially seem from those of typical users of quantum mechanics.

### III. THE “EXPERIMENT”

The specific version of a GHZ experiment considered in this paper is based on that of Mermin<sup>11</sup>: a source emits three spin- $\frac{1}{2}$  particles in the directions of three widely-separated collaborators, Alice, Bob, and Casey, as illustrated in Fig. 1. Each of the collaborators has a detector that will read out the projection of the spin angular momentum of a particle along an axis determined by the orientation of the detector. This is a generalization of the kind of two-particle experiments envisioned by Bell<sup>21</sup> that have been used to test the quantum mechanical issues raised by EPR. In Bell’s original scenario he demonstrated a disagreement between the predictions of a local realist program, like that of EPR, and the predictions of quantum mechanics, but the disagreement manifests itself in terms of an inequality based on statistical measures of the results of many experiments. In GHZ experiments the contrast is more stark: the disagreement is evident in a single run of an experiment. A local-realist program predicts the outcome of a specific experiment, and quantum mechanics predicts something different — there is no need for statistics or inequalities to see the disagreement.

Initial discussion of the experiment will be from the point of view of Alice as the *agent*. Based on her knowledge of the source, and her previous experience using it, Alice is certain about its behavior, and she *assigns* her initial state vector to be the pure entangled state

$$|\psi_0\rangle_A = \frac{1}{\sqrt{2}} \left( |+\rangle_A |+\rangle_B |+\rangle_C - |-\rangle_A |-\rangle_B |-\rangle_C \right), \quad (1)$$

where  $|+\rangle_A$  is a single-particle spin-up state along Alice’s  $z$ -axis, which is the direction from the source to Alice, and  $|-\rangle_A$  is the spin-down state along the same axis. The  $z$ -axes and states for Bob and Casey are defined similarly. Alice’s certainty about her state assignment is reflected in her use of a pure state.<sup>22</sup>

Alice, Bob, and Casey have agreed that in an initial run of the experiment Alice will determine the component of the spin angular momentum of her particle along an axis in the

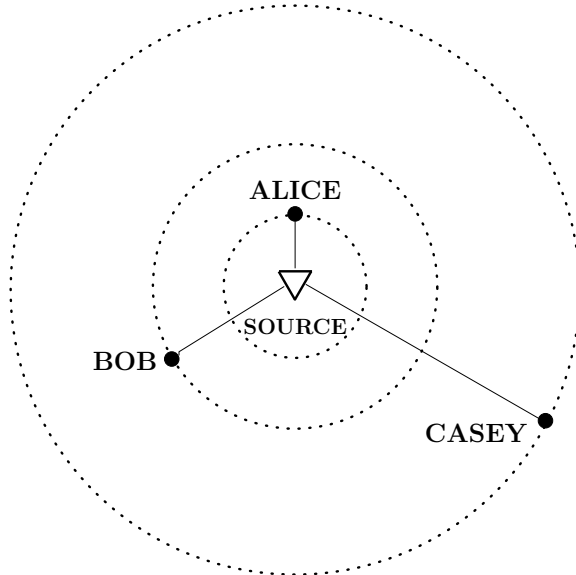


FIG. 1. Configuration of Alice, Bob, and Casey in a GHZ experiment.

plane of the figure, which is her  $x$ -axis, while Bob and Casey will measure the component of spin angular momenta of their particles along axes perpendicular to the plane of the figure, which correspond to their  $y$ -axes.

Before the experiment Alice makes an initial prediction of what she will learn about the readings of the three detectors. Being a skilled user of quantum mechanics, she recognizes that this will be a probabilistic prediction. Alice, being the closest to the source, will be the first to be able to read data about a spin. After doing so, she will update her predictions about what she will learn about data recorded by Bob and Casey. Bob will be the next to read data, after which he will send a message notifying Alice of his result. After eventually getting around to reading her message, Alice will update her predictions again. Those familiar with GHZ experiments will recognize that at this point Alice, to the extent that she trusts the veracity of the message she has read, will know with certainty the result that she will receive in a report from Casey. In the next section I follow Alice's path to such certainty in a sample run of the experiment.

#### IV. BETTING ON OUTCOMES

In order to facilitate making her predictions of the outcomes of the experiment, Alice rewrites her state vector,  $|\psi_0\rangle_A$ , using relationships between the spin- $\frac{1}{2}$  angular momentum

eigenstates:

$$|+z\rangle = \frac{1}{\sqrt{2}} \left( |+x\rangle + |-x\rangle \right) = \frac{1}{\sqrt{2}} \left( |+y\rangle + |-y\rangle \right) \quad (2)$$

and

$$|-z\rangle = \frac{1}{\sqrt{2}} \left( |+x\rangle - |-x\rangle \right) = \frac{i}{\sqrt{2}} \left( -|+y\rangle + |-y\rangle \right). \quad (3)$$

Using Eqs. (2 & 3) in Eq. (1) gives her the alternate expression of her initial state of the particles,

$$\begin{aligned} |\psi_0\rangle_A = \frac{1}{2} \left( & |+x\rangle_A |+y\rangle_B |+y\rangle_C + |+x\rangle_A |-y\rangle_B |-y\rangle_C \right. \\ & \left. + |-x\rangle_A |+y\rangle_B |-y\rangle_C + |-x\rangle_A |-y\rangle_B |+y\rangle_C \right). \end{aligned} \quad (4)$$

Note that in arriving at this result the terms containing the products  $|+x\rangle_A |+y\rangle_B |-y\rangle_C$ ,  $|+x\rangle_A |-y\rangle_B |+y\rangle_C$ ,  $|-x\rangle_A |+y\rangle_B |+y\rangle_C$ , and  $|-x\rangle_A |-y\rangle_B |+y\rangle_C$  have all dropped out due to cancellations.

Having completed this calculation, and before conducting the experiment, Alice is now certain (from Eq.(4)) that when she collects all the data there will only be four possible combinations recorded by Alice, Bob, and Casey respectively:  $(up, up, up)$ ,  $(up, down, down)$ ,  $(down, up, down)$ , and  $(down, down, up)$ . The fact that there must be an odd number of  $up$ 's in any run of the experiment is built in from the beginning by Alice's state assignment. Alice is also certain that each of these outcomes will occur with 25% probability. That means, for example, that she is willing bet that the final result will be  $(down, down, up)$  by paying up to \$0.25 for a ticket that she will be able to redeem for \$1 if the actual result of the experiment is, in fact,  $(down, down, up)$ . Alice is also certain that when she reads the data for her spin she has a 50% chance of finding spin-up, and a 50% chance of finding spin-down.

When Alice reads the output of the detector for her spin, let's say she finds spin-up along her  $x$ -axis. She can now update her probability calculations for the results that she will eventually learn from Bob and Casey. Alice assigns a new state vector by eliminating the final two terms in her state vector of Eq. (4); after normalization Alice's updated state vector is

$$|\psi_1\rangle_A = \frac{1}{\sqrt{2}} |+x\rangle_A \left( |+y\rangle_B |+y\rangle_C + |-y\rangle_B |-y\rangle_C \right). \quad (5)$$

Alice is now certain that when she receives reports from Bob and Casey, she will learn that they detected the same thing, either both *up* or both *down*, and she calculates a 50% probability for each outcome. Alice's interrogation of her particle has eliminated some of the possible outcomes, with an increased probability for those that remain.

Alice notices that the value of *up* read by her detector means that she lost her bet that the outcome would be (*down, down, up*), but this doesn't mean it was a bad bet. It was internally self-consistent, and it was as good as buying a similarly priced ticket betting that the outcome of two successive flips of a believed-to-be-fair coin will be (*tails, tails*).

Some time after Alice looked at her detector, Bob reads the spin projection recorded by his detector, finds it to be *down*, and he sends his message to Alice informing her of his result. Does anything change for Alice at the time Bob reads the output of his detector? No — it's still a good bet, for example, to buy a ticket for up to \$0.50 that will pay \$1.00 if the results of the run will be (*up, up, up*), until the time she reads the message from Bob informing her of the results. It is at this time of that the result recorded by Bob enters her experience.

After reading the message from Bob informing her that his spin was *up*, Alice updates her prediction again. Alice's state vector assigned after receiving this information is

$$|\psi_2\rangle_A = |+\rangle_A |+\rangle_B |+\rangle_C, \quad (6)$$

which leads Alice to predict with certainty the result that she will hear from Casey is *spin-up*. Additional runs of experiments like this confirm Alice's initial prediction (based on Eq. (4)) that there will always be an odd number of spin *up*'s reported.

Was there any wavefunction collapse in this tale? No, just an accumulation of information by the agent Alice that enabled her to sift through the possibilities inherent in the initial entangled state. Bob and Casey could perform analyses similar to Alice's, i.e., we can treat them as agents. There will be times when the three physicists, or three agents, are using different state vectors, but all three will be able to make well-informed bets on the outcome of a run based on their individual experiences.

If Alice, Bob, and Casey had all agreed to report spin components along the *x*-axis, Alice could use a similar procedure to write her initial state vector in the form

$$|\psi_0\rangle_A = \frac{1}{2} \left( |+\rangle_A |+\rangle_B |-\rangle_C + |+\rangle_A |-\rangle_B |+\rangle_C + |-\rangle_A |+\rangle_B |+\rangle_C + |-\rangle_A |-\rangle_B |-\rangle_C \right). \quad (7)$$

In this version of the experiment there are again only four possible data sets for a run, but this time the number of *up*'s detected will be either 0 or 2.

If Alice's detector reads *spin-up* she updates her state vector to

$$|\psi'_1\rangle_A = \frac{1}{\sqrt{2}} |+\rangle_A \left( |+\rangle_B |-\rangle_C + |-\rangle_B |+\rangle_C \right), \quad (8)$$

and she knows that Bob and Casey will report to her that they have detected opposite spin components. Upon reading a message from Bob that his detector also read *spin-up*, Alice is certain that Casey will report to her a reading of *spin-down*.

## V. IMPLICATIONS FOR REALISM

In both of the versions of a GHZ experiment considered in the previous section, the value of a spin component recorded by Casey satisfies the criteria set by Einstein, Podolsky, and Rosen for an *element of reality* because Alice, “without, in any way disturbing a system [Casey's particle], can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity [the direction of the projection of Casey's spin along his *x*-axis], then there exists an element of reality corresponding to this physical quantity.”<sup>8</sup> Casey's spin projections in the examples above are not special, and similar experiments could demonstrate that the other projections along the *x*- and *y*-axes satisfy the same criteria.

Alice and Bob each have friends who want to pursue the consequences of accepting the realism of the spin projections. The friends have seen data from the first kind of experiment in which Alice, Bob, and Casey record data for the spin components along the *x*-, *y*-, and *y*-axes respectively. The realist friends then make predictions for the case in which all of the collaborators detect spins along their *x*-axes. Specifically, they consider the case considered at the conclusion of Section IV in which Alice detects *up* along her *x*-axis, Bob detects *down* along his, and they predict the result for Casey's *x*-axis detection.

From Alice's determination of *up* along her *x*-axis, her friend assigns possible real values to the *y*-components determined by Bob and Casey: Bob and Casey either both determine *up* or both determine *down*. These two possibilities are illustrated graphically in the top two rows in Fig. 2, with assignments for Alice, Bob, and Casey's spins given left to right. Alice's detection is represented by the solid black arrow, and the inferred assumed-real values for Bob's and Casey's spin components are represented by the dashed red arrows.





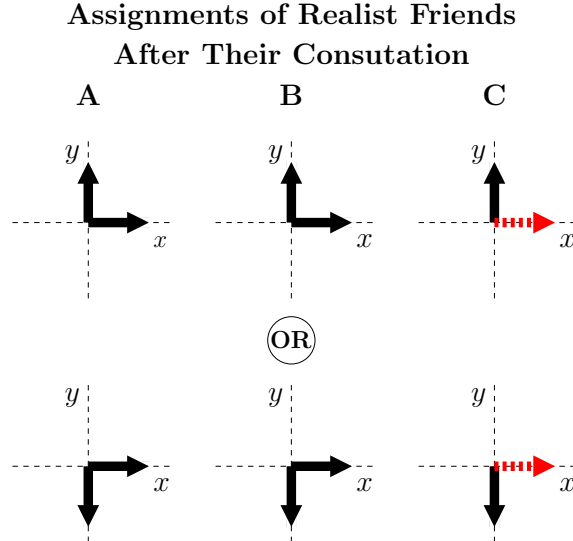


FIG. 3. Assignment of spin components after combining information about the  $x$  and  $y$ -projections of spin, assuming all are elements of reality. The solid black arrows combine the information represented in the top and bottom portions of Fig. 2; the red dashed arrows are inferred from the results of experiments measuring  $s_x$ ,  $s_y$ , and  $s_y$ .

black arrows in Fig. 3. They then use the result from Alice, Bob, and Casey’s first set of experiments that a measurement of  $s_y-s_y-s_x$  always yields an odd number of *spin-ups*, and conclude that Casey must detect *spin-up* for the  $x$ -component for both of their possible assignments; this is represented by the red dashed arrows in Fig. 3.

This is strikingly at odds with the conclusion reached by Alice at the end of Section IV: she predicted that the news she would hear from Casey would be an observation of *down*. When Alice does hear from Casey, she learns that her prediction was correct. (Alice’s confidence in quantum mechanics has been supported by a GHZ experiment using photons prepared in an entangled state.<sup>23</sup>) Where did the realist friends of Alice and Bob go wrong? They assumed that that the projection of the spins along the  $y$ -axes of Alice and Bob had real values, even though no measurements of the  $y$ -components were made, ignoring the reminder of Peres that “unperformed experiments have no results.”<sup>24</sup> Based on the outcome of a single run of the experiment, Alice is prepared to give up the concept of realism as articulated by Einstein, Podolsky, and Rosen.<sup>25</sup>

## VI. DISCUSSION

In the examples in this paper I have articulated a QBist perspective on a well-known quantum mechanics problem. Framing the discussion in this way introduces the notion that state assignments are personal to each agent, and obviates the need for a concept like ‘wavefunction collapse.’ Proponents of QBism argue that the perspective does much more than simply provide a way to re-frame the discussion of standard problems: they argue it opens new horizons for work in quantum foundations<sup>18,19</sup>, as well as new ways to think about science in general.<sup>26</sup> QBism with its roots in personalist Bayesian probability, raises the question of whether it is possible to find a good representation of quantum mechanics that can be framed entirely in terms of probabilities, without the necessity for abstract states, complex amplitudes and Hilbert-space operators. While work in this field is beyond the scope of this paper, it is worth noting the progress in this enterprise,<sup>16,19</sup> and the most general theorem yet in this direction.<sup>27</sup>

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