Math 201

21 October 2008
Second Midterm

NAME (Print!): ____________________

Check one: (1pm): ______
(2pm): ______

Problem	Points	Score
1	20	
2	20	
3	10	
4	30	
5	20	
Total	100	

Problem 1 (20 points): The heat capacity $C(T)$ of a substance is the amount of energy (in joules) required to raise the temperature of 1 gram by 1 degree Celsius above temperature T.

(a) Explain why the energy required to raise the temperature from T_1 to T_2 is the area under the graph of $C(T)$ over $[T_1, T_2]$.

(b) How much energy is required to raise the temperature from 50 to 100 degrees Celsius if $C(T) = 6 + 0.2\sqrt{T}$.

a) Units of $C(T)$ are joules/°C

So $\int_{T_1}^{T_2} \underbrace{C(T)}_{J/°C} \underbrace{dT}_{°C}$ " = " $\int_{T_1}^{T_2} J$

" = total joules from T_1 to T_2

b) $\int_{50}^{100} 6 + 0.2\sqrt{T}\, dT$

$= 6T \Big|_{50}^{100} + \frac{4}{30} T^{3/2} \Big|_{50}^{100} = \ldots$

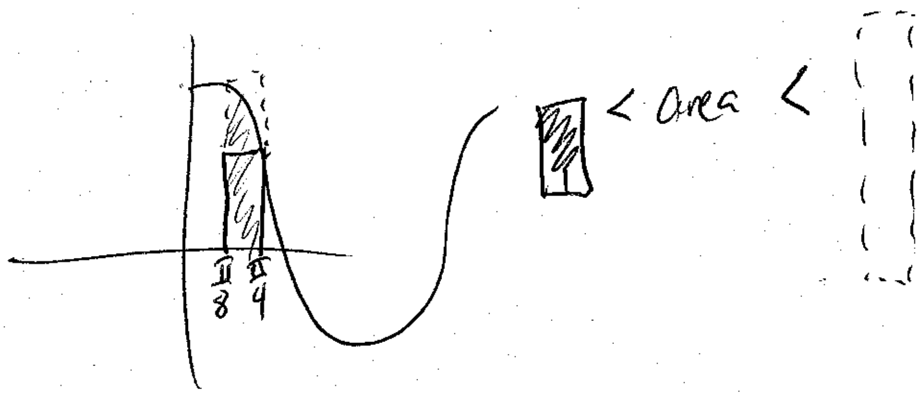
Problem 2 (20 points): Prove that $0.277 \leq \int_{\pi/8}^{\pi/4} \cos x \, dx \leq 0.363$.

$$\sin x \Big|_{\frac{\pi}{8}}^{\frac{\pi}{4}} \approx .35.$$

Therefore

$$0.277 < \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} < 0.363.$$

N.B. I meant to have you ~~seen~~ reason this w/ rectangles



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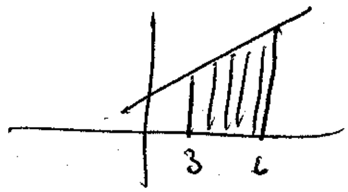
NAME (Print!): ______________________

Check one: (1pm): ______

(2pm): ______

Problem 3 (10 points): Let $f(x) = 2x + 7$ on $[3, 6]$. Find a formula for R_N and find the area under $f(x)$ by taking the limit.

$\cancel{R_N = \frac{3}{N}}$ (



base of each rect : $\frac{3}{N}$

~~height of~~

$R_N = \frac{3}{N}$ (height 1 + height 2 + ... + height N)

$= \frac{3}{N}\left(f\left(3+\frac{3}{N}\right) + \dots + f\left(3+N\cdot\frac{3}{N}\right)\right)$

$= \frac{3}{N}\left(2\left(3+\frac{3}{N}\right)+7+2\left(3+2\cdot\frac{3}{N}\right)+7+\dots+2\left(3+N\frac{3}{N}+7\right)\right)$

$= \frac{3}{N}\left(7N+6N+2\frac{3}{N}+2\cdot 2\cdot\frac{3}{N}+2\cdot 3\cdot\frac{3}{N}+\dots+2\cdot N\cdot\frac{3}{N}\right)$

$= \frac{3}{N}\left(13N+\cancel{20}\frac{6}{N}\left(1+2+\dots+N\right)\right)$

$= \frac{3}{N}\left(13N+\frac{6}{N}\frac{N(N+1)}{2}\right)$

$= \frac{3}{N}\left(13N+3N+3\right) = \frac{3}{N}\left(16N+3\right) = 48+\frac{9}{N}$

$\int_3^6 2x+7\,dx = \lim_{N\to\infty} 48+\frac{9}{N} = \boxed{48}$

Problem 4 (30 points): Compute the following:

(a) $\frac{d}{dx}\int_x^0 \sin^2 t\ dt$ $= -\frac{d}{dx}\int_0^x \sin^2 t\,dt$

$= -\sin^2 x$

(b) $\int_0^{\pi/4} \sec^2\theta\ d\theta$ $= \tan\theta \Big|_0^{\frac{\pi}{4}} = 1-0=1$

(c) $\int \frac{dx}{x\sqrt{\ln x}}$ $= \int \frac{x\ du}{x\sqrt{u}} = \int u^{-1/2}\,du$

$u = \ln x$

$= 2u^{1/2} + C$

$du = \frac{1}{x}\,dx$

$= 2\sqrt{\ln x} + C$

$dx = x\,du$

Problem 5 (20 points): Let $f(x) = x^2 - 5x - 6$ and $F(x) = \int_0^x f(t)\,dt$.

(a) Find the critical points of $F(x)$ and determine whether they are local minima or maxima.

(b) Find the points of inflection of $F(x)$ and determine whether the concavity changes from up to down or vice versa.

By FToC II

$F'(x) = f(x) = x^2 - 5x - 6 = (x-6)(x+1)$

~~FdC~~ critical pts: $6, -1$

when $x = 5$, $f(x) < 0$ @ 6

$x = 7$ $f(x) > 0$ so, a min

$x = 0$ $f(x) < 0$ @ 2 so, a max.

$x = -2$ $f(x) > 0$

$F''(x) = 2x - 5$

$\frac{5}{2}$ possible inflection pt

$F''(3) > 0$ so it goes from down to up.

$F''(2) < 0$ at $\frac{5}{2}$ is an inflection pt.

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