Limit Theorems

Basic Limit Theorems: Assume $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ exist. Then

Sum Law: $\lim_{x\to c} f(x) + g(x) = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$
Constant Multiple Law: $\lim_{x\to c} kf(x) = k \lim_{x\to c} f(x)$
Product Law: $\lim_{x\to c} f(x) \cdot g(x) = (\lim_{x\to c} f(x)) (\lim_{x\to c} g(x))$
Quotient Law: If $\lim_{x\to c} g(x) \neq 0$,

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$$

1: Can the Quotient Law be used to evaluate $\lim_{x\to 0} \sin(x)/x$? Can the Product Law be used to evaluate $\lim_{x\to \pi/2}(x - \pi/2)\tan(x)$? Justify your answer.

2: Assume that $\lim_{x\to 0} f(x)/x = 1$. Which of the following must be true and why?

- $f(0) = 0$
- $\lim_{x\to 0} f(x) = 0$

3: Suppose that $\lim_{h\to 0} g(h) = L$. Explain why $\lim_{h\to 0} g(ah) = L$ for any $a \neq 0$. If we assume instead that $\lim_{h\to 1} g(h) = L$, is it still necessarily true that $\lim_{h\to 0} g(ah) = L$? Verify your answers with the function $f(x) = x^2$.

Squeeze Theorem: Assume that for $x \neq c$ (in some open interval containing c)

$$l(x) \leq f(x) \leq u(x) \text{ and } \lim_{x \to c} l(x) = \lim_{x \to c} u(x) = L$$

Then

$$\lim_{x \to c} f(x) \text{ exists and } \lim_{x \to c} f(x) = L.$$

4: Draw a picture that describes this theorem. And use the picture to decide if you believe the theorem.

5: Use the Squeeze Theorem to show that $\lim_{x \to 0} x \sin(1/x) = 0$. Why couldn't you have used the Product Law?