Limit Theorems

Basic Limit Theorems: Assume $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ exist. Then

Sum Law: $\lim_{x\to c} f(x) + g(x) = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$
Constant Multiple Law: $\lim_{x\to c} kf(x) = k\lim_{x\to c} f(x)$
Product Law: $\lim_{x\to c} f(x)\cdot g(x) = (\lim_{x\to c} f(x))\,(\lim_{x\to c} g(x))$
Quotient Law: If $\lim_{x\to c} g(x) \neq 0$,

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$$

1: Can the Quotient Law be used to evaluate $\lim_{x\to 0}\sin(x)/x$? Can the Product Law be used to evaluate $\lim_{x\to\pi/2}(x-\pi/2)\tan(x)$? Justify your answer.

(a) No, because the denominator ~~goes~~ has a limit of 0 at 0

(b) No, because $\lim_{x\to\frac{\pi}{2}} \tan(x)$ doesn't exist

2: Assume that $\lim_{x\to 0} f(x)/x = 1$. Which of the following must be true and why?

- $f(0) = 0$
- $\lim_{x\to 0} f(x) = 0$

(b) $\lim_{x\to 0} f(x) = \lim_{x\to 0} x \times \frac{f(x)}{x} \overset{\text{product law}}{=} \lim_{x\to 0} x \times \lim_{x\to 0}\frac{f(x)}{x} = 1\cdot 0 = 0$

A counter example for (a): $f(x) = \begin{cases} 5 & \text{if } x=0 \\ x & \text{if } x\neq 0 \end{cases}$

3: Suppose that $\lim_{h\to 0} g(h) = L$. Explain why $\lim_{h\to 0} g(ah) = L$ for any $a\neq 0$. If we assume instead that $\lim_{h\to 1} g(h) = L$, is it still necessarily true that $\lim_{h\to 0} g(ah) = L$? Verify your answers with the function $f(x) = x^2$.

g(ah) is g(h) compressed in towards the line ~~y~~ x=0; the compression around x=0 is symmetric about x=0. So, the y-values of the graph at ~~x~~=0 ~~doesn't~~ don't change. They ~~would~~ could change around x=1, though.

e.g. $f(x) = x^2$ $\quad f(2x) = 4x^2$

$\lim_{x\to 0} x^2 = \lim_{x\to 0} 4x^2 = 0 \neq 4 = \lim_{x\to 1} 4x^2.$

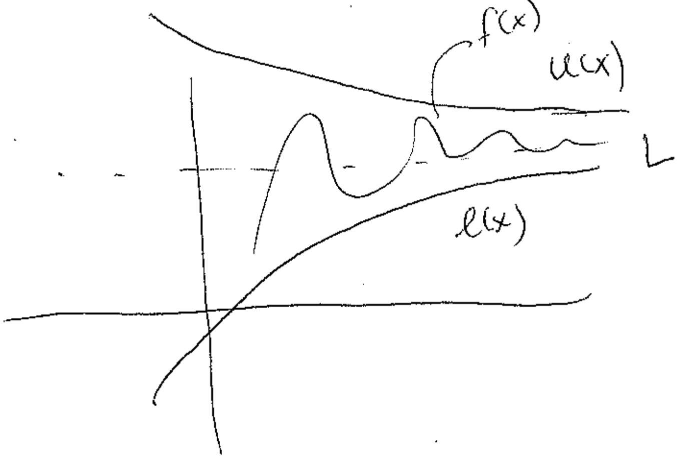
Squeeze Theorem: Assume that for $x \neq c$ (in some open interval containing c)

$$l(x) \leq f(x) \leq u(x) \text{ and } \lim_{x \to c} l(x) = \lim_{x \to c} u(x) = L$$

Then

$$\lim_{x \to c} f(x) \text{ exists and } \lim_{x \to c} f(x) = L.$$

4: Draw a picture that describes this theorem. And use the picture to decide if you believe the theorem.



5: Use the Squeeze Theorem to show that $\lim_{x \to 0} x \sin(1/x) = 0$.

$0 \le |\sin 1/x| \le 1$

$\lim_{x \to 0}$ ~~Let u(x)~~

So $0 \le x|\sin \frac{1}{x}| \le x$ for $x > 0$.

~~Now~~ and $-|x| \le x \sin \frac{1}{x} \le |x|$

Now $\lim_{x \to 0} -|x| = \lim_{x \to 0} |x|$, so by the Squeeze Thm

$\lim_{x \to 0} x \sin \frac{1}{x} = 0.$