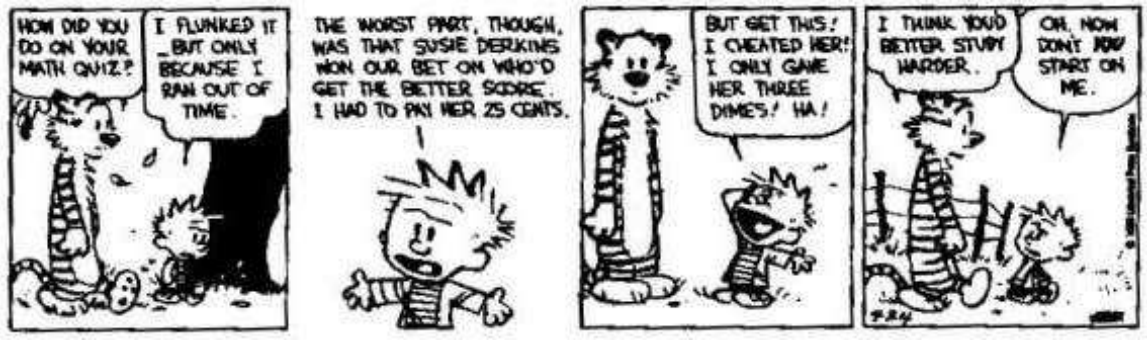
Math 201

23 September 2008
First Midterm

NAME (Print!): _____

Check one: (1pm): _____
(2pm): _____



Problem	Points	Score
1	20	
2	20	
3	30	
4	20	
5	10	
Total	100	

Problem 1 (20 points): According to the Gutenberg-Richter law, the number N of earthquakes worldwide of Richter magnitude M approximately satisfies the relation $\ln N = 16.17 - bM$ for some constant b.

(1) Assuming there are 800 earthquakes of magnitude 5 each year, find b.

(2) Using your b from the first part, how many earthquakes of magnitude 7 occur each year? (use $b = 2$ if you couldn't find an answer to the first part).

Problem 2 (20 points): Let $f(x) = |x|^x$.

(1) Investigate the left-hand and right-hand limits of $f(x)$ as $x \to 0$.
(2) Sketch a graph of $f(x)$ and describe the behavior near 0.
(3) Conclude what the limit is, if it exists, or conclude that the limit as $x \to 0$ doesn't exist.

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Problem 3 (30 points): Find the following limits. For each part, name the laws, theorems and/or rules that you use. If the limit doesn't exist justify your conclusion in some way.

(1) $\lim_{x\to\pi/2} \tan(x)$

(2) $\lim_{x\to 0} \frac{x+3}{x^2-9}$

(3) $\lim_{x\to 2} \frac{\frac{1}{x}-\frac{1}{2}}{x-2}$

(4) $\lim_{x\to 4} \frac{3-\sqrt{x+5}}{x-4}$

Problem 4 (20 points): Each of the following statements is **false**. For each statement sketch the graph of a function that provides a counterexample (assume that the function $f(x)$ is defined on an open interval containing a):

(1) If $\lim_{x\to a} f(x)$ exists then $f(x)$ is continuous at a.

(2) If $f(x)$ has a jump discontinuity at $x = a$, then $f(a)$ equals either $\lim_{x\to a^+} f(x)$ or $\lim_{x\to a^-} f(x)$.

(3) The one sided limits $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ always exist, even if $\lim_{x\to a} f(x)$ doesn't exist.

(4) For (1) above write down a specific $f(x)$ that is a counterexample.

Problem 5 (10 points): Show that the function

$$f(x) = \begin{cases} x^2 \cos(2/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is continuous at 0. Justify your answer by stating what rules/laws/theorems you used.

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