Math 201

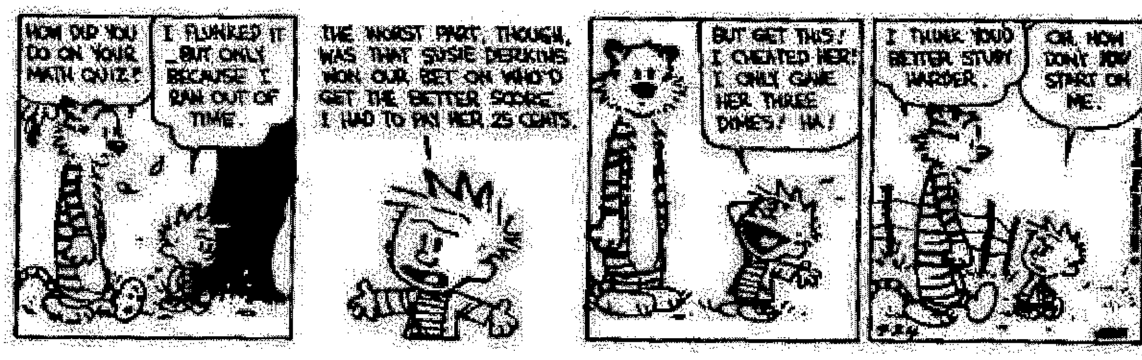
23 September 2008

First Midterm

NAME (Print!): Key

Check one: (1pm): ______

(2pm): ______



Problem	Points	Score
1	20	
2	20	
3	30	
4	20	
5	10	
Total	100	

Problem 1 (20 points): According to the Gutenberg-Richter law, the number N of earthquakes worldwide of Richter magnitude M approximately satisfies the relation $\ln N = 16.17 - bM$ for some constant b.

(1) Assuming there are 800 earthquakes of magnitude 5 each year, find b.

(2) Using your b from the first part, how many earthquakes of magnitude 7 occur each year? (use $b = 2$ if you couldn't find an answer to the first part).

(1) $\ln 800 = 16.17 - 5b$

$$\frac{\ln 800 - 16.17}{-5} = 1.89$$

(2) $\ln N = 16.17 - 1.89 \times 7$

$= 18.9$

~~start~~

so between 18 & 19

Problem 2 (20 points): Let $f(x) = |x|^x$.

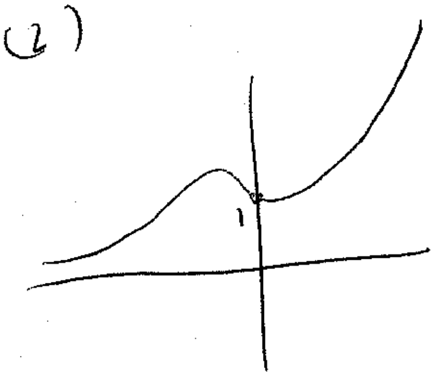
(1) Investigate the left-hand and right-hand limits of $f(x)$ as $x \to 0$.
(2) Sketch a graph of $f(x)$ and describe the behavior near 0.
(3) Conclude what the limit is, if it exists, or conclude that the limit as $x \to 0$ doesn't exist.

(1)

x	f(x)
-0.02	~~.989~~ 1.0125
-0.01	1.0069
.001	.99312
.002	.98765

[illegible]

(2)



(3) $\lim_{x \to 0} |x|^x = 1.$

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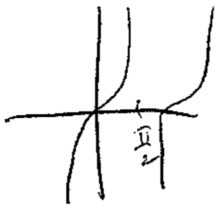
Problem 3 (30 points): Find the following limits. For each part, name the laws, theorems and/or rules that you use. If the limit doesn't exist justify your conclusion in some way.

(1) $\lim_{x\to\pi/2}\tan(x)$



Does not exist since $\lim_{x\to\frac{\pi}{2}^+}\tan x = +\infty$

$\lim_{x\to\frac{\pi}{2}^-}\tan x = -\infty$

(2) $\lim_{x\to 0}\frac{x+3}{x^2-9}$

continuous at 0: $\lim_{x\to 0}\frac{3}{-9} = -\frac{1}{3}$

(3) $\lim_{x\to \cancel{0}\,2}\frac{\frac{1}{x}-\frac{1}{2}}{x-2}$ indeterminate form $\frac{0}{0}$

$\lim_{x\to 2}\frac{\frac{1}{x}-\frac{1}{2}}{x-2} = \lim_{x\to 2}\frac{\frac{2-x}{2x}}{x-2} = \lim_{x\to 2} -\frac{1}{2x}$

$= -\frac{1}{4}$

(4) $\lim_{x\to 4}\frac{3-\sqrt{x+5}}{x-4}$

$\lim_{x\to 4}\left(\frac{3-\sqrt{x+5}}{x-4}\right)\left(\frac{3+\sqrt{x+5}}{3+\sqrt{x+5}}\right) = \lim_{x\to 4}\frac{9-(x+5)}{(x-4)(3+\sqrt{x+5})}$

$= \lim_{x\to 4}\frac{-x+4}{(x-4)(3+\sqrt{x+5})}$

$= \lim_{x\to 4}\frac{-1}{3+\sqrt{x+5}}$

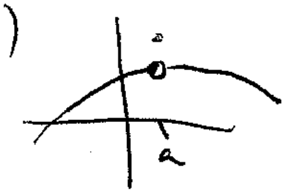
$= -\frac{1}{6}$

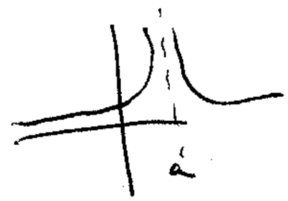
Problem 4 (20 points): Each of the following statements is **false**. For each statement sketch the graph of a function that provides a counterexample (assume that the function $f(x)$ is defined on an open interval containing a):

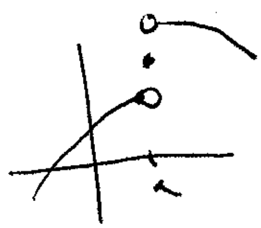
(1) If $\lim_{x\to a} f(x)$ exists then $f(x)$ is continuous at a.

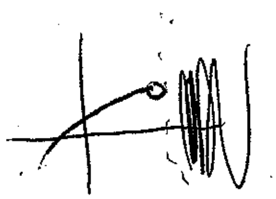
(2) If $f(x)$ has a jump discontinuity at $x = a$, then $f(a)$ equals either $\lim_{x\to a^+} f(x)$ or $\lim_{x\to a^-} f(x)$.

(3) The one sided limits $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ always exist, even if $\lim_{x\to a} f(x)$ doesn't exist.

(1) 

or 

(2) 

(3) 

(4) For (1) above write down a specific $f(x)$ that is a counterexample.

$$f(x) = \begin{cases} 1 & \text{when } x \neq a \\ 2 & \text{when } x = a \end{cases} \qquad f(x) = \frac{1}{(x-a)^2}$$

Problem 5 (10 points): Show that the function

$$f(x) = \begin{cases} x^2\cos(2/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is continuous at 0. Justify your answer by stating what rules/laws/theorems you used.

I need to show

(1) $f(0)$ is defined $\leftarrow$ $f(0)=0$ ✓

(2) $\lim_{x\to 0} f(x)$ exists

and

(3) $f(0) = \lim_{x\to 0} f(x)$.

since $\cos\frac{2}{x} \geq -1$.

$$\lim_{x\to 0} -x^2 \leq \lim_{x\to 0} x^2\left(\cos\frac{2}{x}\right) \leq \lim_{x\to 0} x^2$$ since $\cos\frac{2}{x} < 1$

$\lim_{x\to 0} \pm x^2 = 0$ since $\pm x^2$ is continuous. So

$$0 \leq \lim_{x\to 0} x^2\left(\cos\left(\frac{2}{x}\right)\right) \leq 0$$

By the Squeeze theorem $\lim_{x\to 0} x^2\left(\cos\frac{2}{x}\right) = 0$

Therefore $\lim_{x\to 0} f(x)$ exists and equals $f(0)$ so

f is continuous at 0.

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