Math 201

23 September 2008
First Midterm

NAME (Print!): KEY

Check one: (1pm): _____
(2pm): _____

Problem	Points	Score
1	20	
2	20	
3	30	
4	20	
5	10	
Total	100	

Problem 1 (20 points): The number of cells in a petri dish is given by an exponential: $c(t) = ae^{bt}$. At time 0 minutes, there were 5,000,000 cells and for the entire experiment 45% of them are dying each minute.

(1) Find a and b.

(2) Starting at what time will there be less that 1,000 cells. (Use $a = 100$ and $b = 0.5$ if you couldn't get the first part).

(1) $c(0) = ae^0$

$5000000 = a$

at $t=1$, $0.45 \times 5{,}000{,}000$ remain so

$2\,250000 = 5000000\, e^{b1}$

$\frac{2250000}{5000000} = e^{b1}$

$0.45 = e^{b1}$

$\ln 0.45 = b$

$b = -.798$

(2) $1000 = 5000000\, e^{-.8t}$

$t = \frac{\ln\left(\frac{1000}{5000000}\right)}{-0.8} = 10.6$ minutes

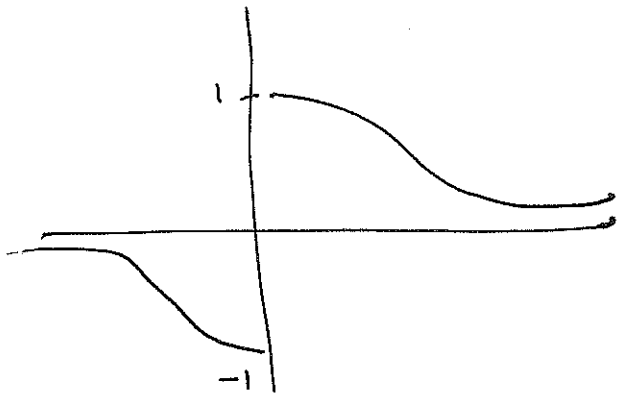
Problem 2 (20 points): The function

$$f(x) = \frac{2^{1/x} - 2^{-1/x}}{2^{1/x} + 2^{-1/x}}$$

is defined for $x \neq 0$.

(1) Investigate the left-hand and right-hand limits of $f(x)$ as $x \to 0$.

(2) Sketch a graph of $f(x)$ and describe the behavior near 0.



$\lim_{x \to 0} f(x)$ does not exist as

$\lim_{x \to 0^+} f(x) = 1$

$\lim_{x \to 0^-} f(x) = -1.$

Problem 3 (30 points): Find the following limits. For each part, name the laws, theorems and/or rules that you use.

(1) $\lim_{x\to\pi/4} \frac{\sin x-\cos x}{\tan x-1}$

can't plug in $\frac{\pi}{4}$ $\quad \frac{\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}}{1-1}$

$\Rightarrow$ algebra

$$\frac{\sin x-\cos x}{\frac{\sin x}{\cos x}-1} = \frac{\sin x-\cos x}{\frac{\sin x-\cos x}{\cos x}} = \cos x$$

$$\lim_{x\to\frac{\pi}{4}} \cos x = \frac{\pi}{4}$$

(2) $\lim_{x\to 0} \tan^{-1}(e^x)$

$\tan^{-1}$ is continuous since $\tan$ is. e^x is continuous

So $\lim_{x\to 0} \tan^{-1}(e^x) = \tan^{-1}(1) = \frac{\pi}{4}$

(3) $\lim_{x\to 0^+} \sqrt{x}e^{\cos(\pi/x)}$ can't plug in 0; no algebra seems helpful. So squeeze!

$-1 < \cos\frac{\pi}{x} < 1$

$$\frac{1}{e} < e^{\cos\frac{\pi}{x}} < e$$

$$\frac{\sqrt{x}}{e} < \sqrt{x}\, e^{\cos\frac{\pi}{x}} < e\sqrt{x}$$

$\lim_{x\to 0^+} \sqrt{x}\, e^{\cos\frac{\pi}{x}} = 0$, by the ST since

$\lim_{x\to 0^+} \frac{\sqrt{x}}{e} = \lim_{x\to 0^+} e\sqrt{x} = 0.$

Problem 4 (20 points): Answer the following two questions:

(1) According to the Law of Continuity, if $f(x)$ and $g(x)$ are continuous at c, then $f(x) + g(x)$ is continuous at c. Suppose that $f(x)$ and $g(x)$ are discontinuous at c. Is it true that $f(x) + g(x)$ is discontinuous at c? If not, give a counterexample.

No. $\frac{1}{x}$ and $\frac{-1}{x}$ are discontinuous at 0

but their sum is continuous at 0.

(2) Is it true that if $f(x)$ and $g(x)$ are continuous at $x = a$, then $f(x)/g(x)$ is continuous at $x = a$? If not, give a counterexample.

Let $a = 0$.

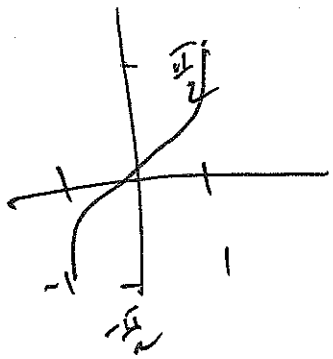
$g(x) = x$
$f(x) = 1$] continuous everywhere

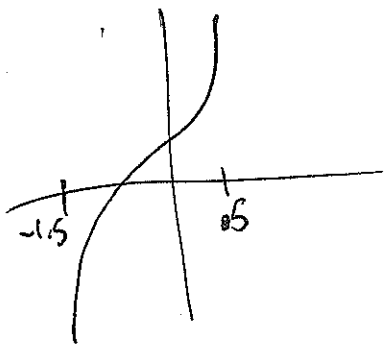
$\frac{f(x)}{g(x)} = \frac{1}{x}$ ← not continuous at 0.

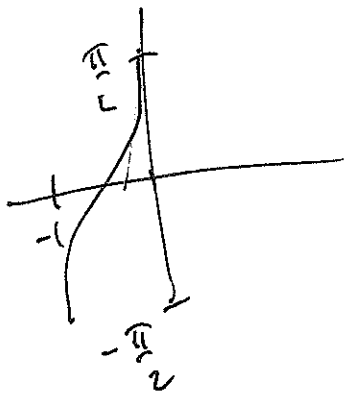
Problem 5 (10 points): Find the domain and range for the following

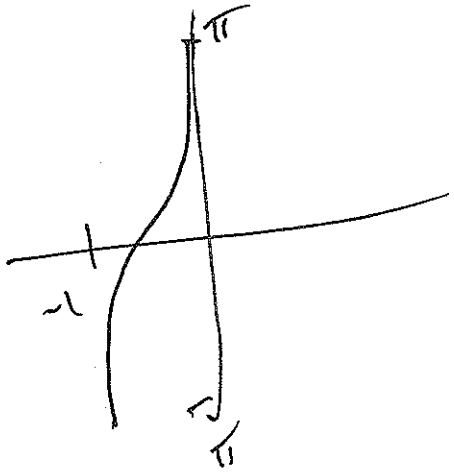
$$f(x) = 2\sin^{-1}(2x+1).$$

Sketch the function.









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