Rates of Change

(1): The position of a particle is given by the equation

$$s = f(t) = t^3 - 6t^2 + 9t$$

where t is measured in seconds and s in meters.

(a) Find the velocity at time t.
(b) When is the particle at rest?
(c) When is the particle moving to the left? To the right?

(a) $\frac{ds}{dt} = 3t^2 - 12t + 3$

(b) $\frac{ds}{dt} = 3(t-3)(t-1)$ @ $t = 1, 3$

(c) $\frac{ds}{dt} > 0$ when $t > 3$ or $t < 1$

$\frac{ds}{dt} < 0$ when $1 < t < 3$

(2): A spherical snowball starts at 70 cm. As it melts, its radius decreases at a constant rate of 2 cm per minute. How fast is the volume decreasing half an hour later?

$V = \frac{4\pi}{3} R^3$ $\quad R(t) = 70 - 2t$

$$\frac{dV}{dt} = \frac{dV}{dR}\frac{dR}{dt}$$

$$= \frac{4\pi}{\cancel{3}} R^2 (-2)$$

$$= 4\pi (70-2t)^2 (-2)$$

@ $t = 30$

$$\left.\frac{dV}{dt}\right|_{30} = 4\pi \cdot 100 (-2)$$

$$= -800\pi$$

(3): Coroners estimate time of death using the rule that a body cools about 2 degrees Fahrenheit in the first hour a 1 degree for each additional hour. Assuming an air temperature of 68 degrees and a living body temperature is 98.6 degrees, the temperature $T(t)$ of a body in degrees Fahrenheit is given by

$$T(t) = 68 + 30.6e^{-kt}.$$

(a) For what value of k will the body cool by two degrees in the first hour?
(b) Using the value of k found above, after how many hours will the temperature of the body be decreasing by 1 degree an hour?
(c) Compare what the model predicts and what the coroner's rule predicts after 24 hours.

(a) $k = -\ln \dfrac{96.6 - 68}{30.6} = 0.067$

(b) need $\dfrac{dT}{dt} = -1$ ..

$$\frac{dT}{dt} = -k\, 30.6 e^{-kt}$$

$$-1 = -0.067 \cdot 30.6\, e^{-0.067t}$$

$$\Rightarrow t = 10.3$$

(c) Corononers $\Rightarrow$ 98.6 after 24 hours is 25° cooler $\Rightarrow$ 73.6

~~(c)~~ model $T(24) = 68 + 30.6 e^{-0.067 \cdot 24}$

$= 74.1$.