Math 201

18 November 2008

Third Midterm

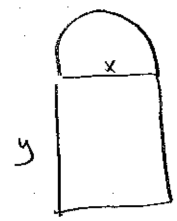
NAME (Print!): ____________________

Check one: (1pm): ______

(2pm): ______

Problem	Points	Score
1	20	
2	20	
3	30	
4	20	
5	10	
Total	100	

Problem 1 (20 points): A window has the shape of a rectangle surmounted by a semicircle. (Thus the diameter of the semicircle is equal to the width of the rectangle.) If the perimeter of the window is 30 ft, find the dimensions of the window so that the greatest amount of light is admitted. Finding the min or max from your calculator's graph isn't enough.



$$x + 2y + \frac{\pi x}{2} = 30 \Rightarrow \frac{30 - \frac{\pi}{2}x - x}{2} = y$$

$$15 - \frac{\pi}{4}x - \frac{x}{2} = y$$

$$A(x,y) = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$$

$$A(x) = x\left(15 - \frac{\pi}{4}x - \frac{x}{2}\right) + \frac{1}{8}\pi x^2$$

$$= 15x - \frac{\pi}{4}x^2 - \frac{x^2}{2} + \frac{1}{8}\pi x^2$$

for $0 \leq x \leq \frac{30}{1+\frac{\pi}{2}}$

x will be smallest when x=0 and largest when y=0

$$x + \frac{\pi}{2}x = 30$$

$$x = \left(\frac{30}{1+\frac{\pi}{2}}\right)$$

1 Find critical points

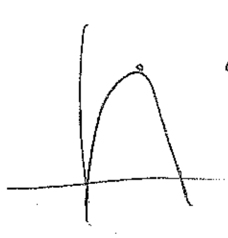
$$15 - \frac{\pi}{2}x - x + \frac{1}{4}\pi x = 0$$

$$15 = \left(\frac{\pi}{2} + 1 - \frac{\pi}{4}\right)x$$

$$\frac{15}{\frac{\pi}{2} + 1 - \frac{\pi}{4}} = x$$

$$8.401 \approx \frac{60}{\pi + 4} = x$$

checked on graphing calculator



8.6 → 62.9

2 $A(0) = 0$

$A\left(\frac{30}{1+\frac{\pi}{2}}\right) \approx 33.477$

$A(8.401) \approx 63.039$ ✓

Problem 2 (20 points): Using Newton's method, find, correct to six decimal places, the root of the equation $\cos x - x = 0$. Also explain why there is only one root.

$\cos x - x = 0$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{\cos(x_n) - x_n}{-\sin(x_n) - 1}$$

let $x_0 = 1$

$$x_1 = 1 + \frac{\cos(1) - 1}{\sin(1) + 1} \approx .750$$

$x_2 \approx$ ~~.84~~ $.739113$

~~$x_3 \approx .81$~~

~~.817358~~

$x_3 \approx .739085$

$x_4 \approx .739085 \leftarrow$ approximate root root

Since $f'(x) < 0$ for all x, $f(x)$ is always decreasing so it must have one root at most.

Root also checked on graphing calculator

Problem 3 (30 points): Graph the functions $f(x) = xe^x$. Be sure to indicate clearly the

- domain of the function
- vertical and horizontal asymptotes (or tell me why there aren't any)
- minima and maxima (or tell me why there aren't any)
- inflection points (or tell me why there aren't any)

Domain : $\mathbb{R}$

Intercepts: $x=0 \Rightarrow y=0 \quad \Rightarrow$ Goes through $(0,0)$

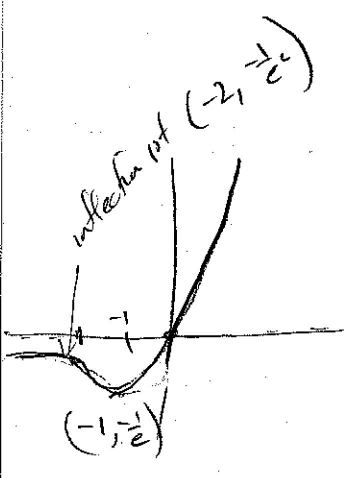
$y=0 \Rightarrow x=0$.

Asymptotes

$\lim_{x\to\infty} xe^x = +\infty$

$\lim_{x\to-\infty} xe^x = 0$ since $\lim_{x\to-\infty} xe^x = -\lim_{x\to\infty} \frac{x}{e^x}$ "$\frac{\infty}{\infty}$"

$= -\lim_{x\to\infty} \frac{1}{e^x} = 0$.



First derivative

$f'(x) = xe^x + e^x = (x+1)e^x$

$x<-1$	$-$
$x>-1$	$+$

$\leftarrow$ min at $x=-1$

Second derivative

$f''(x) = e^x + (x+1)e^x = (x+2)e^x$

	f''
$x<-2$	$-$
$x>-2$	$+$

$\leftarrow$ inflection pt at $x=-2$

Problem 4 (20 points): Find the following

- all functions $g(x)$ so that $g'(x) = 4\sin x + \frac{2x^5 - \sqrt{x}}{x}$.

antiderivative :

$\cancel{-4\cos x}$

$g'(x) = 4\sin x + 2x^4 - x^{-\frac{1}{2}}$

$\Downarrow$

$G(x) = -4\cos x + \frac{2}{5}x^5 - 2x^{\frac{1}{2}} + C$

check

$G'(x) = 4\sin x + 2x^4 - x^{-\frac{1}{2}}$. ✓

- $f(x)$ is $f'(x) = e^x + 20(1+x^2)^{-1}$ and $f(0) = 1$.

$f(x) = e^x + 20\arctan x + C$

$f(0) = 1 + 20\arctan 0 + C$

$C = 1$

$f(x) = e^x + 20\arctan x + 1$.

Problem 5 (10 points): Prove that

$$\lim_{x\to 3} \frac{x}{5} + 2 = \frac{13}{5}$$

using the ε, δ definition of the limit.

① Find δ as a function of ε

$\cancel{\lim_{x\to 3}} \left| \frac{x}{5} + 2 - \frac{13}{5} \right| < \varepsilon$

$\cancel{\lim_{x\to 3}} \left| \frac{x}{5} - \frac{3}{5} \right| < \varepsilon$

$\cancel{\lim_{x\to 3}} |x-3| < 5\varepsilon$

~~② Let $\delta = 5\varepsilon$~~

② Check $\delta = 5\varepsilon$ works.

I want to show $|x-3| < \delta \Rightarrow \left| \frac{x}{5} + 2 - \frac{13}{5} \right| < \varepsilon$.

Now $|x-3| < \delta$

$|x-3| < 5\varepsilon \quad (\delta = 5\varepsilon)$

$\left| \frac{x}{5} - \frac{3}{5} \right| < \varepsilon \quad (\div)$

$\left| \frac{x}{5} + 2 - \frac{13}{5} \right| < \varepsilon \quad \left(2 - \frac{13}{5} = -\frac{3}{5} \right)$. $\square$