Measurement of Newton's Gravitational Constant

PHYS 310

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Abstract

In this experiment you will make a measurement of the constant G that appears in Newton's gravitational force law. This is the least well known of the fundamental constants. You will use a Cavendish torsion balance to compare the very small gravitational force between two lead balls to another known force. The method you will use is essentially the same as that used by Cavendish in the 18th century, and is still the basis for more refined measurements that are being done to this day.

I. INTRODUCTION

In this experiment you will measure the gravitational force between two small masses by comparing it to another small force. The magnitude of the gravitational force between two masses M and m separated by a distance r is

$$F_{\text{grav}} = G\frac{Mm}{r^2}. \tag{1}$$

Recall that this law is valid for spherically symmetric mass distributions as well as for point masses. For such mass distributions the distance r is the separation between the centers of the spheres.

The "other" small force is the restoring force on a small mass caused by the twisting of a torsional balance. The balance consists of a $\perp$-shaped suspension to which two small balls of mass m are attached. This assembly is suspended on a fine ($0.01 \times 0.15\,\text{mm}$) bronze wire attached to the top of the apparatus.

The basic idea of the experiment is very simple, and is illustrated in Fig. 1, a view looking down on the apparatus from above. When the large balls are rotated into their fully-clockwise orientation (Position 1), the suspended rod with the small balls is deflected counter-clockwise, and when the large balls are rotated into their fully counter-clockwise position (Position 2), the suspended rod rotates clockwise. If the suspended rod lies along the center-line of the box when the large balls are in the neutral position (Position 3), then the equilibrium deflection angles in Positions 1 and 3 will have the same magnitude θ.

The magnitude of the torque supplied by the twisting of the wire in the torsion balance is directly proportional to the angle of twist (for small angles), i.e.,

$$\tau_{\text{torsion}} = \kappa\theta, \tag{2}$$

where κ is the constant of proportionality. To determine the value of G we will use the fact that in equilibrium we must have

$$\tau_{\text{grav}} = \tau_{\text{torsion}}. \tag{3}$$

II. PRELIMINARY DETERMINATION OF G

You will first determine a value of G making some simplifying assumptions. In this section you may assume that both balls are point masses; you may assume that the suspended rod

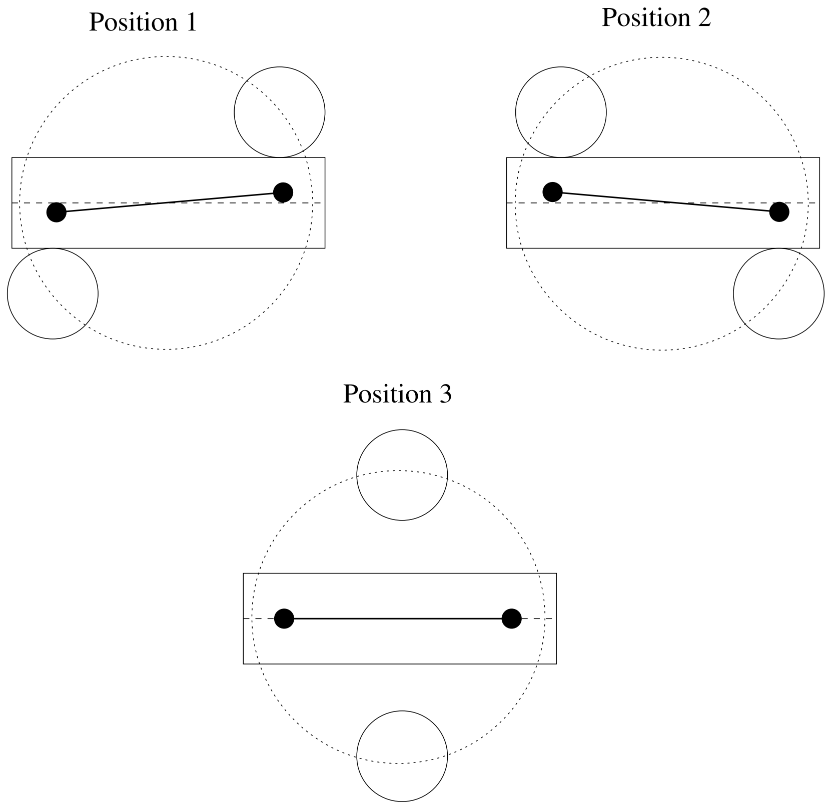

FIG. 1: Overhead view of the masses in the Cavendish balance.

lies along the center-line of the box when the large balls are in the neutral position (Position 3 in Fig. 1); you may assume that the force of attraction between a large mass and the small mass on the far end of the suspended rod is negligible; you may assume that the mass of the rod joining the small masses is negligible; you may assume that the balls are uniform spheres; and you may assume that the deflection of the suspended rod is small enough that the mass separation b is approximately the distance between the center of the large mass and the center of the box.

A. Determination of rotation angle θ

In order to measure the small rotation angle θ you can measure the deflection of a light beam from the mirror which is attached to the suspended assembly holding the small balls. You should convince yourself that the deflection angle of the laser is twice the rotation angle of the mirror. In the geometry illustrated in Fig. 2 the rotation angle is determined by the relationship

$$\tan 2\theta = \frac{x}{L}, \tag{4}$$

where x is the distance the laser spot moves on the "screen" when the mirror rotates by θ, and L is the distance from the mirror to the screen. For small angles, this gives

$$\theta \simeq \frac{x}{2L}. \tag{5}$$

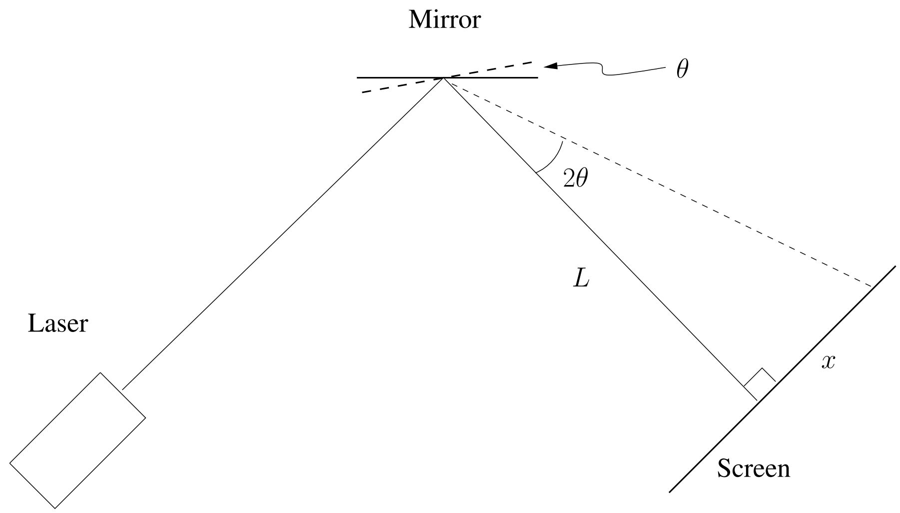

FIG. 2: Geometry for determination of rotation angle.

You should set up the apparatus so that a laser hits the mirror of the apparatus in such a way that the reflected beam forms a spot near the center of a horizontal 2-meter stick when the balls are in Position 3.

B. Determination of torsion constant κ

It is difficult to measure directly the proportionality constant κ in Eq. (2), but a linear restoring torque will result in oscillations about the equilibrium position whose period depends on the value of κ.

Problem 1 Show that the torsion constant κ is given by

$$\kappa = \frac{4\pi^2 I}{T^2}, \tag{6}$$

where I is the moment of inertia of the small balls about the center of rotation, and T is the period of free oscillations of the small balls when no large balls are present.

Gently set the torsion pendulum in motion. You can do this by first establishing equilibrium with the balls in Position 1 or 2, and then quickly but carefully removing the large balls. Record the position of the laser spot on the meter stick every 30 seconds for several periods of oscillation. The motion will be slow, so data collection may take over half an hour. This data can be fit using a computer to determine the period of oscillation, and thus the torsion constant κ.

C. Preliminary Formula for G

We need to derive a formula for G in terms of experimentally measurable quantities. What can we measure?

- The mass of the large ball M.
- The radius of the large ball R.
- The width of the box c.
- The deflection of the laser spot on the meter stick. Let's agree to call s the linear distance on the meter stick between the equilibrium position of the spot when the large balls are in Position 1 and the equilibrium position when the balls are in position 2. (This obviates the need to determine the equilibrium position of the spot when the balls are in the neutral Position 3.)
- The position of the meter stick relative to the apparatus.
- The period of free oscillation of the small balls.

There are a few other quantities that we could measure in principle, but the measurement of these would involve the dismantling of the apparatus. For these we will use the values supplied by the manufacturer:

- The mass of the small ball, $38.3 \pm 0.2\,\mathrm{g}$,
- The radius of the small ball, $r = 0.953\,\mathrm{cm}$,
- The length of the rod to which the small balls are attached, $d = 10\,\mathrm{cm}$.

Problem 2 Derive an approximate expression for the gravitational torque on the suspended rod in terms of the measured quantities listed above. For this preliminary treatment you may make the simplifying assumptions discussed in Section II. Use Eq. (3) to determine an approximate expression for G in terms of directly measured quantities.

Make the necessary measurements to make a preliminary determination of the value of G. Make sure that this value is the right order of magnitude before proceeding.

III. FINAL DETERMINATION OF G

Before making a final determination of the value of G you will analyze the relative sizes of the various uncertainties in your preliminary measurement. You will also estimate the magnitude of various systematic errors. After assessing these errors, you will decide how to improve your preliminary determination of G, and assign an uncertainty to your final value.

1. Determine the magnitude of the errors in your preliminary determination of G due

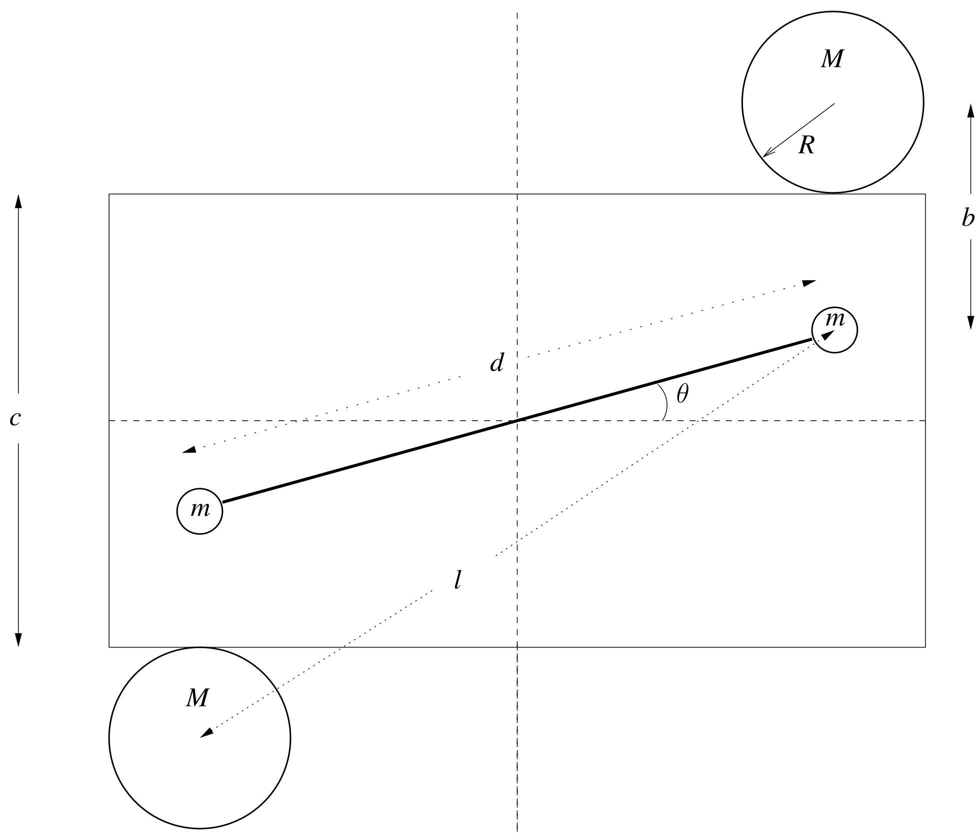

FIG. 3: Schematic diagram of masses.

to the random uncertainties in the measured quantities. For example, there is an uncertainty ΔM in the measured value of the large mass, and this contributes an uncertainty ΔG_M to your value of G.

2. Make a list of systematic uncertainties in your preliminary measurement and discuss this list with your instructor. This will involve thinking about any assumptions that you made in determining your preliminary value. Make a quantitative estimate of the magnitude of the uncertainty introduced by each of these systematic errors; this may involve some significant calculation.

3. Identify the largest sources of uncertainty in your experiment, and decide on a set of reasonable ways to improve both your experimental measurements and your theoretical formula.

4. Determine your best value for G including an appropriately determined uncertainty. Your notebook should include a table with entries for each of the directly measured quantities, and the uncertainty associated with each measured quantity.