A Rotation Curve for Milky Way Galaxy

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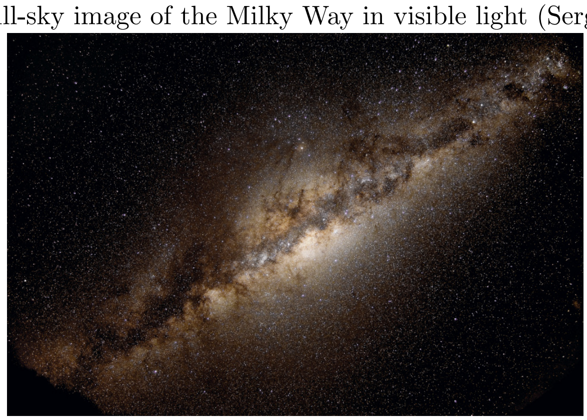
1 The Milky Way Galaxy

We live inside a fairly large spiral galaxy, but because of our location within it, it's very hard to see this structure. Most of us think of the Milky Way as a stripe of stars and nebulosity that stretches across the sky.

Figure 1: All-sky image of the Milky Way in visible light (Serge Brunier).



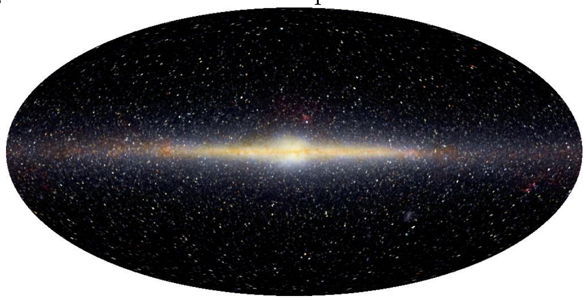
Indeed, this "splash of milk" look is how our galaxy got its name. It isn't hard to infer that this star- and dust-filled stripe across the sky delineates a large, flat disk-like structure, viewed more or less edge-on. However, when looking in visible light, you're really only seeing a small fraction of the galaxy. The light from the really distant parts is absorbed by dust long before it reaches your eyes.

Luckily, we can observe the galaxy using other wavelength light. Infrared light is particularly good for this purpose, as dust does not absorb this type of light as readily. Consequently, an image in infrared light shows the full extent and edge-on structure of our galaxy:

Here, the edge-on nature of the galactic disk is quite evident, as is the galaxy's bulgy center. Except for the stars in that bulge, nearly all of the stars, gas, and dust are confined to a very thin ($h \sim 200$ pc[1]) and very extensive

[1] A parsec (pc) is equal to 3.26 light years, or 3.086×10^{16} m.

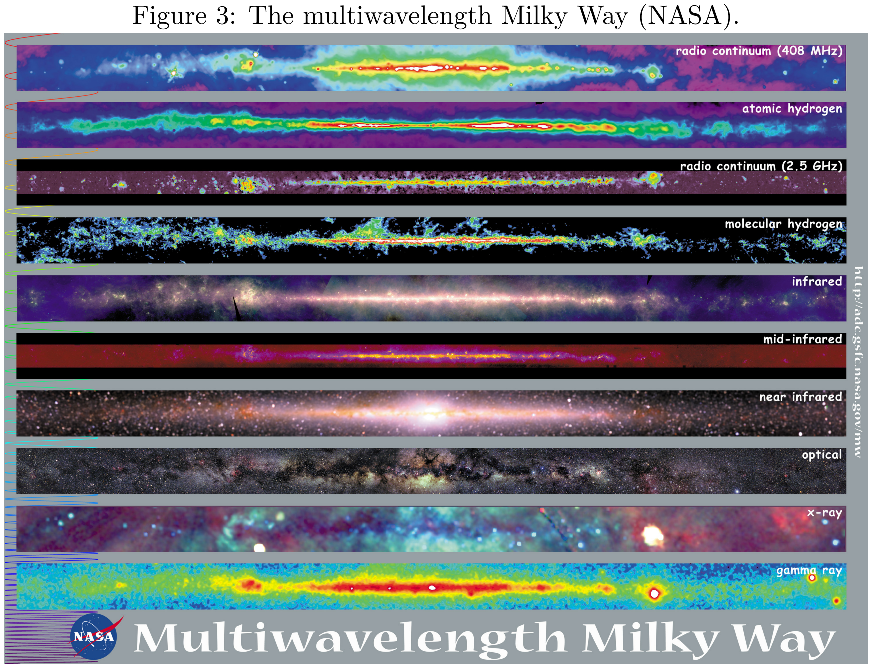
Figure 2: An infrared image of the Milky Way from the COBE satellite. Note that this image is rotated relative to the previous one.



($r \sim 10,000$ pc) disk that (as we will see) rotates about its center.

In fact, one can observe the emission from the galaxy over a wide range of wavelengths, and each type of observation yields additional information about its structure (Figure 3). Note in particular, the observations labeled "atomic hydrogen" and "molecular hydrogen." These measurements were made using radio astronomy techniques, and they show that the distribution of interstellar hydrogen is tightly confined to the galactic disk. In this project, we will use a radio telescope to measure the emission from the atomic hydrogen in the disk.

Figure 3: The multiwavelength Milky Way (NASA).

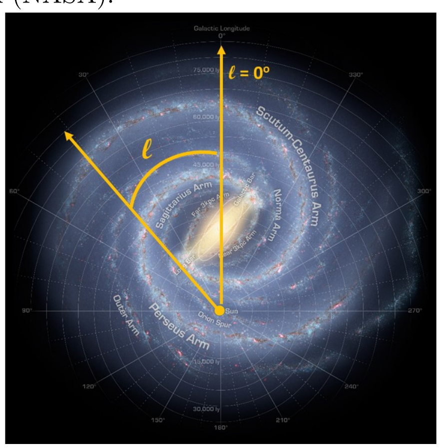


2 The Galactic Coordinate system

Astronomers who study the Milky Way have created a coordinate system to refer to observation directions relative to the galaxy's center and disk. We could have just used the standard equatorial (i.e., R.A. and Dec.) coordinate system, but because the galaxy has a well-defined disk whose center is also well-defined, a galactic coordinate system is more intuitive and easier to use.

The galactic coordinate system defines directions in terms of *longitude (l)* and *latitude (b)*. The center of the coordinate system is *us* (i.e., not the center of the galaxy), and we define the direction toward the center of the galaxy as *(l,b) = (0,0)*. The plane defined by the galactic disk is assigned $b = 0^o$, so that directions passing through the disk are distinguished by different values for l (Figure 4).

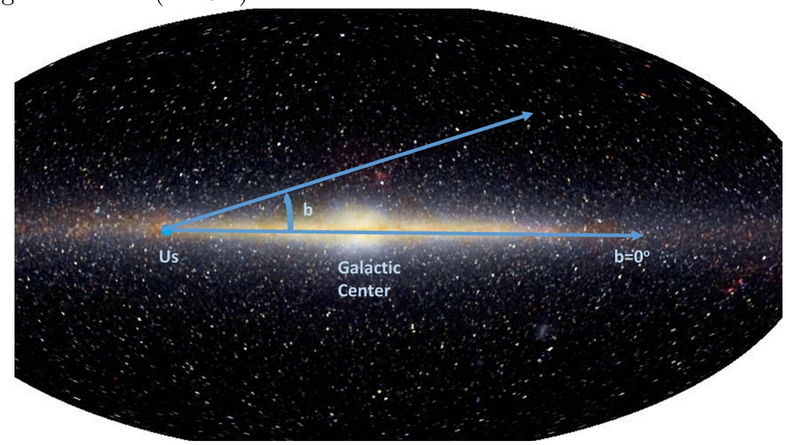
Figure 4: Directions of galactic longitude superimposed on a model for the Milky Way galactic disk (NASA).



Note that the inner part of the galaxy (i.e., directions that pass through parts of the disk closer to the center than us) corresponds to galactic longitudes l = *0* - 90^o and 270 - 360^o, while directions defined by $90^o > l > 270^o$ pass through the more sparsely populated outer disk.

Nonzero values of galactic latitude correspond to directions that pass above or below the galactic disk (Figure 5). Since most of the stars, gas, and dust are confined to the galaxy's disk, these lines of sight tend to be dominated by very nearby objects.

Figure 5: Directions of galactic latitude superimposed on a model for the Milky Way galactic disk (NASA).



3 Spectral Line Radio Emission from Atomic Hydrogen

We will measure the emission from the cold hydrogen gas that can be found throughout the galactic disk (see Figure 3). The vast majority of this gas is far from any stars or other sources of ionizing radiation, so it is in atomic form in its ground electronic state. Therefore, there's no way for the atom to make a transition to a lower electronic state, and so it can't produce a photon.

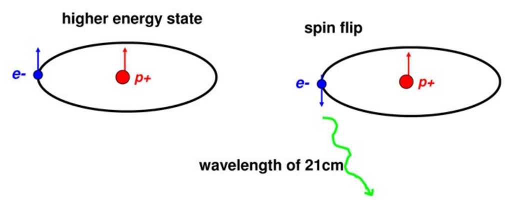
Or can it? Quantum Mechanics tell us that there are two ways an electron can occupy the ground state: spin-up, and spin-down. Atoms with electrons in either of these two states have very nearly the same energy, but not exactly. The difference comes from the interactions between the magnetic moments of the nucleus (a proton) and the electron. Both particles are charged and they have intrinsic spin, so just like a spinning macroscopic charge, they produce a dipole magnetic field. When the dipoles are aligned (e.g., with both "norths" up), the energy is slightly higher than when the dipoles are mis-aligned. (Think of placing two bar magnets side by side; they would "want" to be matched up so that one "north" pole is next to the other's "south" pole.)

The difference in energy between these two states is $\Delta E = 5.9 \times 10^{-6}$ eV, so a "spin-flip" transition between these two states releases a photon of this energy. The wavelength of this photon is

$$\lambda = \frac{hc}{\Delta E} = 21 \text{ cm}$$

This wavelength corresponds to a frequency of $f = 1.4204 \times 10^{9}$ Hz, or 1420.4 MHz – firmly in the radio frequency regime. Note that this is spectral line emission – *only* photons with this frequency (or wavelength) are emitted

Figure 6: Cartoon depiction of the hydrogen atom in its higher energy state (left) and lower energy state(right) due to interactions between the magnetic moments of the proton and electron.(NASA)



by the atomic hydrogen. This is commonly called "H I" emission – the H represents hydrogen, and the Roman numeral I indicates that the gas is not ionized (for example, H II emission comes from ionized hydrogen atoms, and Fe XIV emission comes from iron atoms that have been stripped of 13 electrons).

Cold atomic hydrogen is spread evenly through our galactic disk, and though the densities are very low (n $\sim 10-100$ cm^{-3}), the sight lines through our disk are very large ($\sim 10^{22}$ cm), so there's a lot of atomic hydrogen along most lines of sight. Most of it is excited into the higher energy level (either by collisions, or from photo-excitation from distant stars or the cosmic background radiation). As a result, there is a measurable 21 cm photon flux coming from all parts of the galactic disk, and we can use observations of this emission to investigate the structure and dynamics of the disk.

4 The Doppler Effect and Measuring line-of-sight Velocities from Spectral Line Observations

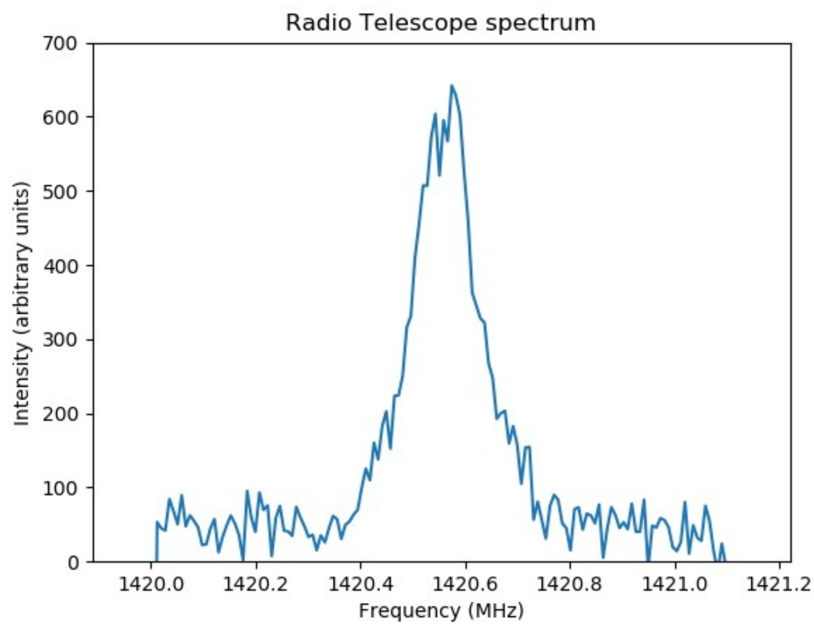
Radio telescope observations of H I emission from our galactic disk show that this emission is spread over a range of frequencies around 1420.4 MHz because the clouds emitting this radiation are moving relative to us. The Doppler shift describes the relationship between the observed frequency f_{obs} and the relative line of sight velocity v_{los} between the emitter and detector

$$\frac{f_o - f_{obs}}{f_o} = \frac{v_{los}}{c}$$

where f_o is the *rest frequency* of the H I emission (f_o = 1420.406 MHz) and c is the speed of light. The above expression is actually an approximation of the fully relativistic Doppler relation; for the speeds we will encounter in our galaxy the approximation is sufficiently valid.

In astronomy, the velocity convention is that velocities away from you are *positive*, while velocities toward you are *negative*. This means that if you detect H I emission with a frequency greater than f_o, the hydrogen atoms responsible for the emission are moving toward you, and if you detect H I emission with a frequency less than f_o, the emitting hydrogen atoms are moving away from you. Typically, one detects emission spread across a small range in frequencies, indicating that the emitters along the line of sight are moving with a range of different velocities. An example of the emission is shown below.

Figure 7: An example H I emission spectrum. All of the detected emission (above a noise level of about 50 in this example) comes from atomic hydrogen spin flips somewhere along the sampled line of sight.



Exercise #1: The peak of the emission spectrum in the above plot occurs at a frequency of 1420.575 MHz. Determine the line-of-sight speed (relative to the telescope) of the atoms producing this emission. Are they moving toward, or away, from the telescope?

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5 H I Emission from the Rotating Galactic Disk

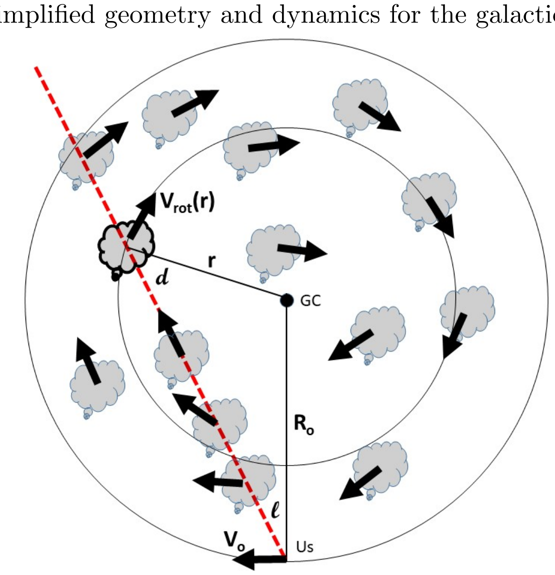
The disk of our galaxy contains a lot of atomic hydrogen, and so it produces a lot of H I emission. The gas isn't distributed evenly throughout the disk, but instead clumped into a large number of giant gas clouds, each many tens of parsecs in size. Using this spectral line emission and the Doppler relation, we can determine the speed of these gas clouds as they orbit the center of the galaxy. If this gas is mixed up with all of the stars, dust, and other components of the galactic disk (a very good assumption), then the velocities that we measure describe the motion of **all** of the material in the galactic disk. That is, we use the H I emission as a *tracer* of dynamics of the total mass in the galactic disk.

Measuring the speed of the disk material is complicated by two factors: 1) using the Doppler relation, we only observe the line of sight component of the difference in velocity between the H I gas and us[2], and 2) when we make an observation of the galactic disk, we detect H I emission from every gas cloud along the line of sight throughout the galactic disk. Therefore, we need to make a few simplifying assumptions regarding the motion of the material in the galactic disk. We will create *model* for the dynamics of the disk, and then use our observations to constrain that model. Here are the important assumptions that go into the model:

- all material in the galactic disk orbits the center of the galaxy in circular orbits of unchanging speed,
- the rotation velocity $v_{rot}(r)$ is a function of galactocentric radius r only,
- we also live in the galactic disk, and our portion of the galactic disk is located a distance $r(Us) = R_o = 8300$ pc from the galactic center, moving at a speed $v_{rot}(R_o) = V_o = 250000$ m/s (Schonrich, 2012).

A diagram of this simplified model for the galactic disk is shown below.

Figure 8: Simplified geometry and dynamics for the galactic disk.



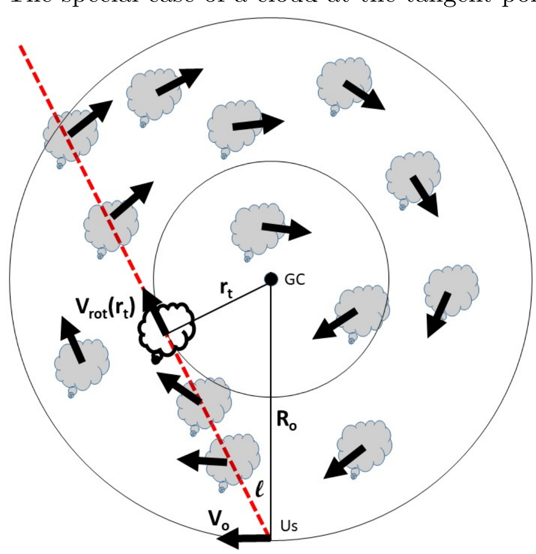
[2]Note that *we* are moving around the center of the galaxy, too!

This is a “face-on” view of the disk, with cartoon H I clouds orbiting around the galactic center (GC). We are located at the position marked “Us.” Note that our disk model assumes that there are many more clouds than depicted here, and that their circular velocities (indicated by the arrows) will vary with distance from the galactic center. In our cartoon, an observation made at galactic longitude l (as indicated by the red dashed line) intercepts five clouds along the line of sight. We will measure the frequencies of the H I emission from each of those clouds to be different from the rest frequency. For each cloud, you will measure a frequency shifted by the component of that cloud's velocity along the line of sight **minus** the component of your velocity along the line of sight (remember, you're moving, too!). We call this the “line of sight” velocity, or v_{los}.

Exercise #2: Consider the outlined cloud in the figure above. It lies a distance r from the center of the galaxy, and therefore has a rotational speed $v_{rot}(r)$. Determine an expression for the line-of-sight velocity v_{los} that you would measure for this cloud, observed from our vantage point. Your expression should be a function of $v_{rot}(r), R_o, r, v_o$, and l. (Hint: the law of sines might come in handy here.)

For every line of sight through the disk, there is a “special” location where the line of sight is tangent to the orbital circle (we call this, appropriately enough, the *tangent point*). At this distance r_t from the galactic center, the rotational velocity $v_{rot}(r_t)$ is directly along the line of sight. In the figure below, the tangent point cloud is shaded white.

Figure 9: The special case of a cloud at the tangent point.



For many physically realizable rotation curves (i.e., $v_{rot}(r)$ vs. r), the tangent point cloud has the largest v_{los} of all clouds along that line of sight. This turns out to be very useful, because the fastest moving cloud (i.e., the most Doppler-shifted H I emission) is easy to identify, even in complex H I spectra.

Exercise #3: Determine an expression for v_{los} for a cloud at the tangent point. Then compare this expression with the general expression developed in Exercise #2, and show that the tangent point cloud is the fastest moving cloud along the line of sight, provided that $v_{rot}(r)$ increases no faster than linearly with r.

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6 Detecting and Measuring Radio Wavelength Emission

Our radio telescope consists of a large steel parabolic reflector (the "dish"), which reflects and focuses light onto a receiver mounted at the focus of the paraboloid. The diffraction limit for an aperture of this size determines the range of directions from the sky (the "beam") from which radio waves will be focused onto the receiver. Unlike visible wavelength telescopes, which use cameras to image a portion of the sky, our radio telescope is a "single pixel" device in that it measures the total power. Thus, the output from a single observation is a spectrum of all the detected radio waves from the beam (e. g., Figure 7). If one wishes to make an image with this telescope, it involves assembling the image point-by-point by moving the telescope and collecting data at each point.

The radio waves are focused onto a "feed," which for our telescope is simply an exposed steel wire. The oscillating electric field of the incoming electromagnetic radiation[3] excites the electrons in this conducting material, producing a fluctuating current which can be amplified and detected electronically. The electronic signal is then processed through a spectrometer to determine the detected power as a function of frequency. Additional details on the detection process can be found in the "Bucknell University Small Radio Telescope Handbook" and references therein.

Radio astronomers use somewhat odd units to describe the radio wave brightness of detected emission. Since radio waves have a long wavelength, the radio portion of the electromagnetic spectrum is typically in the Rayleigh-Jeans regimes for most thermal emitters. Thus one can write the Planck function intensity as

$$I(\lambda, T) = \frac{2c}{\lambda^4} k_B T$$

where λ is the wavelength, c is the speed of light, and k_B is Boltzmann's constant. Radio astronomers turn this equation around and define the *brightness*

[3] In radio astronomy, it is much more intuitive to think of light as a train of superposed electromagnetic waves of a range of frequencies, rather than a stream of photons with different energies.

temperature as

$$T_B = \frac{I\lambda^4}{2k_B c}$$

even for sources that are not thermal emitters. That is, the brightness temperature is simply an expression of the measured intensity, expressed in units of temperature. Therefore, the H I spectra that you obtain from the telescope will have temperature units on their y-axes. Sigh. Just another set of weird units that astronomers use to make their work just a little more opaque...

7 Making Sense of all those Velocities – v_{obs}, v_{los}, V_{LSR}, etc.

The output data from a radio telescope observation consists of a spectrum – the brightness of the emission (in temperature units) as a function of frequency. We can tune the radio telescope to collect data over a specific range of frequencies. The maximum range for our radio telescope is about 1.2 MHz, so if we wish to observe the spectrum around the 1420.406 MHz H I line, we can detect emission from a frequency range of about $1419.8 > f > 1421.0$ MHz. The spectrum is divided into 156 "channels," each of which has a width of ~ 7.8 kHz, and so the output consists of 156 separate brightness measurements, each at a slightly different frequency.

We will assume that all of the emission we detect is H I emission, and that it's not all detected with a frequency equal to 1420.406 MHz due to the relative motion between the emitters (i.e., the hydrogen gas) and the detector (our radio telescope). Therefore, we can convert this frequency scale into a velocity scale, using the Doppler relation. Our spectrum will then consist of brightness as a function of observed velocity (v_{obs}).

This is *almost* what we need to begin our process of measuring the dynamics of the Milky Way galaxy. However, there's one more complication. Our radio telescope sits on a rotating planet that also orbits around a star, which moves mostly in a circle around the center of the Milky Way, but not quite. As a result, the vector velocity of the telescope, relative to the center of the galaxy, is changing rapidly with time, and the component of that velocity along any line of sight is not only a function of time, but also a function of the direction you happen to point the telescope.

This means, for example, that if you were to observe a cloud of atomic hydrogen on two different occasions, and try to calculate the velocity of the cloud (relative to the telescope) by measuring the Doppler shift of the emission, you would likely come up with two different velocities! The cloud hasn't changed speeds between the two observations – *the telescope has!*

If we really want to study the motions of the gas clouds in the galaxy (or any objects anywhere in space), we need to remove the contribution of the local motions, such as the rotation of the Earth, its orbital motion around the Sun, and even the Sun's motion through space. Astronomers do this by defining a

"Local Standard of Rest" (LSR). You can think of the LSR as a velocity frame that's independent of any of our solar system motions, but it's not entirely stationary. The LSR frame is the average velocity of all the stars in our local region, and, in the "circular motion" model of the galaxy we described in Section 5, the LSR moves around the center of our galaxy in a circular orbit with speed V_o.

If we reference all of our measurements to the LSR, then we will always measure the same velocities for all those galactic gas clouds. We simply need to correct the velocities we measure by taking out the motion of the telescope relative to the LSR. Luckily for us, the radio telescope calculates that correction for every single observation – it's called the V_{LSR}. Thus, to obtain the v_{los} in the context of the circular-motion galaxy model[4], subtract the V_{LSR} from the observed velocity v_{obs}:

$$v_{los} = v_{obs} - V_{LSR}$$

[4]You can see here that v_{los} is sort of a misnomer. What we really mean is the line of sight velocity between the observed gas cloud and the telescope *if it were stationary relative to the LSR.*