13

INTERFERENCE OF TWO BEAMS OF LIGHT

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F. A. Jenkins & H. E. White
(McGraw-Hill, 1976)

It was stated at the beginning of the last chapter that two beams of light can be made to cross each other without either one producing any modification of the other after it passes beyond the region of crossing. In this sense the two beams do not interfere with each other. However, in the region of crossing, where both beams are acting at once, we are led to expect from the considerations of the preceding chapter that the resultant amplitude and intensity may be very different from the sum of those contributed by the two beams acting separately. This modification of intensity obtained by the superposition of two or more beams of light we call *interference*. If the resultant intensity is zero or in general less than we expect from the separate intensities, we have *destructive* interference, while if it is greater, we have *constructive* interference. The phenomenon in its simpler aspects is rather difficult to observe, because of the very short wavelength of light, and therefore was not recognized as such before 1800, when the corpuscular theory of light was predominant. The first man successfully to demonstrate the interference of light, and thus establish its wave character, was Thomas Young. In order to understand his crucial experiment performed in 1801, we must first consider the application to light of an important principle which holds for any type of wave motion.

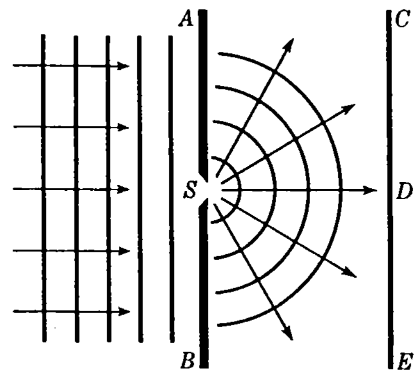


FIGURE 13A
Diffraction of waves passing through a small aperture.

13.1 HUYGENS' PRINCIPLE

When waves pass through an aperture or past the edge of an obstacle, they always spread to some extent into the region which is not directly exposed to the oncoming waves. This phenomenon is called *diffraction.* In order to explain this bending of light, Huygens nearly three centuries ago proposed the rule that *each point on a wave front may be regarded as a new source of waves.** This principle has very far-reaching applications and will be used later in discussing the diffraction of light, but we shall consider here only a very simple proof of its correctness. In Fig. 13A let a set of plane waves approach the barrier AB from the left, and let the barrier contain an opening S of width somewhat smaller than the wavelength. At all points except S the waves will be either reflected or absorbed, but S will be free to produce a disturbance behind the screen. It is found experimentally, in agreement with the above principle, that the waves spread out from S in the form of semicircles.

Huygens' principle as shown in Fig. 13A can be illustrated very successfully with water waves. An arc lamp on the floor, with a glass-bottomed tray or tank above it, will cast shadows of waves on a white ceiling. A vibrating strip of metal or a wire fastened to one prong of a tuning fork of low frequency will serve as a source of waves at one end of the tray. If an electrically driven tuning fork is used, the waves can be made apparently to stand still by placing a slotted disk on the shaft of a motor in front of the arc lamp. The disk is set rotating with the same frequency as the tuning fork to give the stroboscopic effect. This experiment can be performed for a fairly large audience and is well worth doing. Descriptions of diffraction experiments in light will be given in Chap. 15.

If the experiment in Fig. 13A is performed with light, one would naturally expect, from the fact that light generally travels in straight lines, that merely a narrow patch of light would appear at D. However, if the slit is made very narrow, an ap-

* The "waves" envisioned by Huygens were not continuous trains but a series of random pulses. Furthermore, he supposed the secondary waves to be effective only at the point of tangency to their common envelope, thus denying the possibility of diffraction. The correct application of the principle was first made by Fresnel, more than a century later.

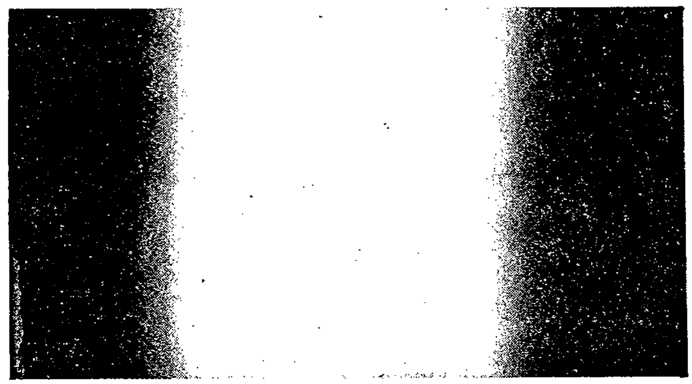


FIGURE 13B
Photograph of the diffraction of light from a slit of width 0.001 mm.

preciable broadening of this patch is observed, its breadth increasing as the slit is narrowed further. This is remarkable evidence that light does not always travel in straight lines and that waves on passing through a narrow opening spread out into a continuous fan of light rays. When the screen *CE* is replaced by a photographic plate, a picture like the one shown in Fig. 13B is obtained. The light is most intense in the forward direction, but its intensity decreases slowly as the angle increases. If the slit is small compared with the wavelength of light, the intensity does not come to zero even when the angle of observation becomes 90°. While this brief introduction to Huygens' principle will be sufficient for understanding the interference phenomena we are to discuss, we shall return in Chaps. 15 and 18 to a more detailed consideration of diffraction at a single opening.

13.2 YOUNG'S EXPERIMENT

The original experiment performed by Young is shown schematically in Fig. 13C. Sunlight was first allowed to pass through a pinhole S and then, at a considerable distance away, through two pinholes S_1 and S_2. The two sets of spherical waves emerging from the two holes interfered with each other in such a way as to form a symmetrical pattern of varying intensity on the screen AC. Since this early experiment was performed, it has been found convenient to replace the pinholes by narrow slits and to use a source giving monochromatic light, i.e., light of a single wavelength. In place of spherical wave fronts we now have cylindrical wave fronts, represented equally well in two dimensions by the same Fig. 13C. If the circular lines represent crests of waves, the intersections of any two lines represent the arrival at those points of two waves with the same phase or with phases differing by a multiple of 2π. Such points are therefore those of maximum disturbance or brightness. A close examination of the light on the screen will reveal evenly spaced light and dark bands or fringes, similar to those shown in Fig. 13D. Such photographs are obtained by replacing the screen AC of Fig. 13C by a photographic plate.

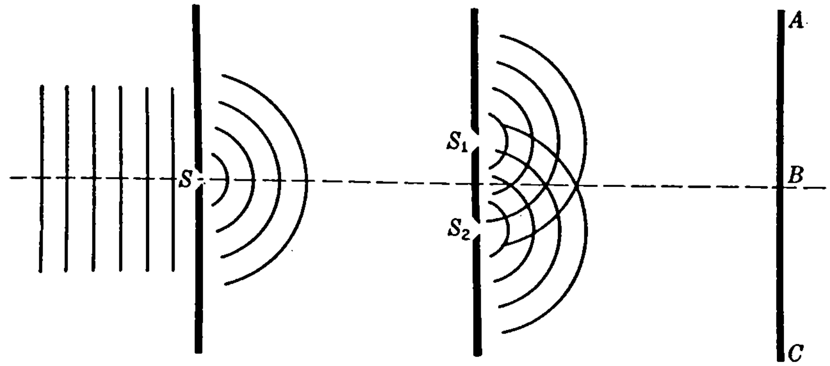


FIGURE 13C
Experimental arrangement for Young's double-slit experiment.

A very simple demonstration of Young's experiment can be accomplished in the laboratory or lecture room by setting up a single-filament lamp L (Fig. 13E) at the front of the room. The straight vertical filament S acts as the source and first slit. Double slits for each observer can be easily made from small photographic plates about 1 to 2 in. square. The slits are made in the photographic emulsion by drawing the point of a penknife across the plate, guided by a straightedge. The plates need not be developed or blackened but can be used as they are. The lamp is now viewed by holding the double slit D close to the eye E and looking at the lamp filament. If the slits are close together, for example, 0.2 mm apart, they give widely spaced fringes, whereas slits farther apart, for example, 1.0 mm, give very narrow fringes. A piece of red glass F placed adjacent to and above another of green glass in front of the lamp will show that the red waves produce wider fringes than the green, which we shall see is due to their greater wavelength.

Frequently one wishes to perform accurate experiments by using more nearly monochromatic light than that obtained by white light and a red or green glass filter. Perhaps the most convenient method is to use the sodium arc now available on the market or a mercury arc plus a filter to isolate the green line, $\lambda5461$. A suitable filter consists of a combination of didymium glass, to absorb the yellow lines, and a light yellow glass, to absorb the blue and violet lines.

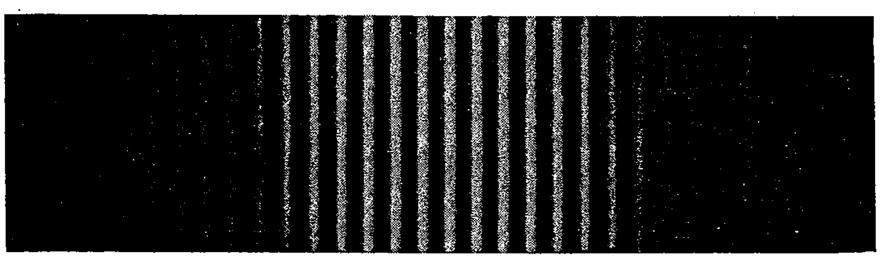


FIGURE 13D
Interference fringes produced by a double slit using the arrangement shown in Fig. 13C.

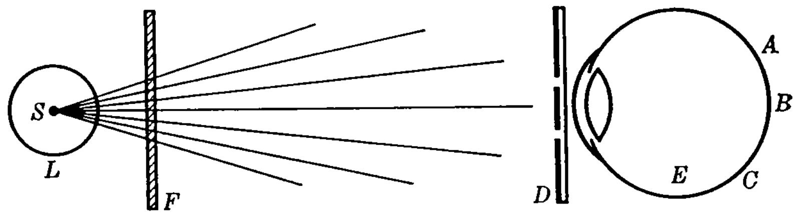


FIGURE 13E
Simple method for observing interference fringes.

13.3 INTERFERENCE FRINGES FROM A DOUBLE SOURCE

We shall now derive an equation for the intensity at any point P on the screen (Fig. 13F) and investigate the spacing of the interference fringes. Two waves arrive at P, having traversed different distances S_2P and S_1P. Hence they are superimposed with a phase difference given by

$$\delta = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda}(S_2P - S_1P) \tag{13a}$$

It is assumed that the waves start out from S_1 and S_2 in the same phase, because these slits were taken to be equidistant from the source slit S. Furthermore, the amplitudes are practically the same if (as is usually the case) S_1 and S_2 are of equal width and very close together. The problem of finding the resultant intensity at P therefore reduces to that discussed in Sec. 12.1, where we considered the addition of two simple harmonic motions of the same frequency and amplitude, but of phase difference δ. The intensity was given by Eq. (12g) as

$$I \approx A^2 = 4a^2 \cos^2 \frac{\delta}{2} \tag{13b}$$

where a is the amplitude of the separate waves and A that of their resultant.

It now remains to evaluate the phase difference in terms of the distance x on the screen from the central point P_0, the separation d of the two slits, and the distance D from the slits to the screen. The corresponding path difference is the distance S_2A in Fig. 13F, where the dashed line S_1A has been drawn to make S_1 and A equidistant from P. As Young's experiment is usually performed, D is some thousand times larger than d or x. Hence the angles θ and θ' are very small and practically equal. Under these conditions, S_1AS_2 may be regarded as a right triangle, and the path difference becomes $d \sin \theta' \approx d \sin \theta$. To the same approximation, we may set the sine of the angle equal to the tangent, so that $\sin \theta \approx x/D$. With these assumptions, we obtain

$$\Delta = d \sin \theta = d\frac{x}{D} \tag{13c}$$

This is the value of the path difference to be substituted in Eq. (13a) to obtain the phase difference δ. Now Eq. (13b) for the intensity has maximum values equal to

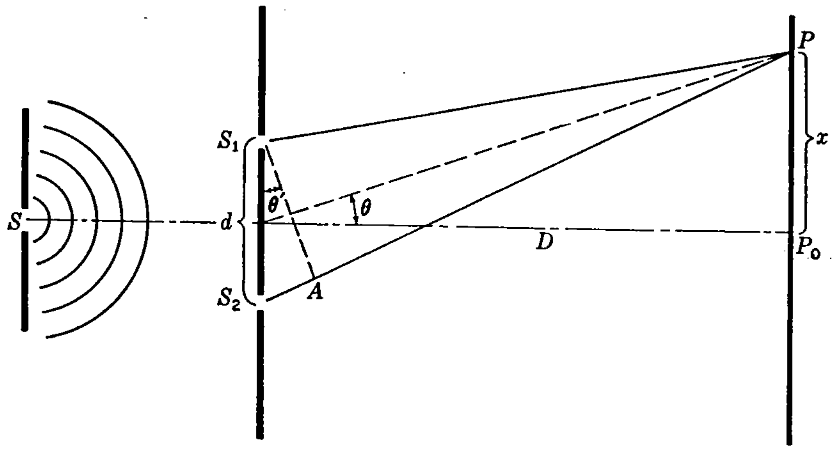


FIGURE 13F
Path difference in Young's experiment.

$4a^2$ whenever δ is an integral multiple of 2π, and according to Eq. (13a) this will occur when the path difference is an integral multiple of λ. Hence we have

$$\frac{xd}{D} = 0, \lambda, 2\lambda, 3\lambda, \ldots = m\lambda$$

or

$$x = m\lambda \frac{D}{d} \qquad \textit{Bright fringes} \qquad (13d)$$

The minimum value of the intensity is zero, and this occurs when $\delta = \pi, 3\pi, 5\pi, \ldots$. For these points

$$\frac{xd}{D} = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots = \left(m + \frac{1}{2}\right)\lambda$$

or

$$x = \left(m + \frac{1}{2}\right)\lambda \frac{D}{d} \qquad \textit{Dark fringes} \qquad (13e)$$

The whole number m, which characterizes a particular bright fringe, is called the *order* of interference. Thus the fringes with $m = 0, 1, 2, \ldots$ are called the *zero, first, second,* etc., *orders.*

According to these equations the distance on the screen between two successive fringes, which is obtained by changing m by unity in either Eq. (13d) or (13e), is constant and equal to $\lambda D/d$. Not only is this equality of spacing verified by measurement of an interference pattern such as Fig. 13D, but one also finds by experiment that its magnitude *is directly proportional to the slit-screen distance D, inversely proportional to the separation of the slits d, and directly proportional to the wavelength λ.* Knowledge of the spacing of these fringes thus gives us a direct determination of λ in terms of known quantities.

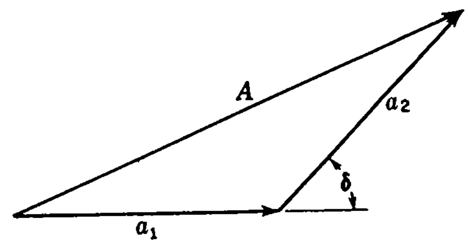


FIGURE 13G
The composition of two waves of the same frequency and amplitude but different phase.

These maxima and minima of intensity exist throughout the space behind the slits. A lens is not required to produce them, although they are usually so fine that a magnifier or eyepiece must be used to see them. Because of the approximations made in deriving Eq. (13c), careful measurements would show that, particularly in the region near the slits, the fringe spacing departs from the simple linear dependence required by Eq. (13d). A section of the fringe system in the plane of the paper of Fig. 13C, instead of consisting of a system of straight lines radiating from the midpoint between the slits, is actually a set of hyperbolas. The hyperbola, being the curve for which the difference in the distance from two fixed points is constant, obviously fits the condition for a given fringe, namely, the constancy of the path difference. Although this deviation from linearity may become important with sound and other waves, it is usually negligible when the wavelengths are as short as those of light.

13.4 INTENSITY DISTRIBUTION IN THE FRINGE SYSTEM

To find the intensity on the screen at points between the maxima, we may apply the vector method of compounding amplitudes described in Sec. 12.2 and illustrated for the present case in Fig. 13G. For the maxima, the angle δ is zero, and the component amplitudes a_1 and a_2 are parallel, so that if they are equal, the resultant $A = 2a$. For the minima, a_1 and a_2 are in opposite directions, and $A = 0$. In general, for any value of δ, A is the closing side of the triangle. The value of A^2, which measures the intensity, is then given by Eq. (13*b*) and varies according to $\cos^2 (\delta/2)$. In Fig. 13H the solid curve represents a plot of the intensity against the phase difference.

In concluding our discussion of these fringes, one question of fundamental importance should be considered. If the two beams of light arrive at a point on the screen exactly out of phase, they interfere destructively and the resultant intensity is zero. One may well ask what becomes of the *energy* of the two beams, since the law of conservation of energy tells us that energy cannot be destroyed. The answer to this question is that the energy which apparently disappears at the minima actually is still present at the maxima, where the intensity is greater than would be produced by the two beams acting separately. In other words, the energy is not destroyed but merely redistributed in the interference pattern. The *average* intensity on the screen is exactly that which would exist in the absence of interference. Thus, as shown in Fig. 13H, the intensity in the interference pattern varies between $4a^2$ and zero. Now each beam acting separately would contribute a^2, and so without interference we would have a uniform intensity of $2a^2$, as indicated by the broken line. To obtain

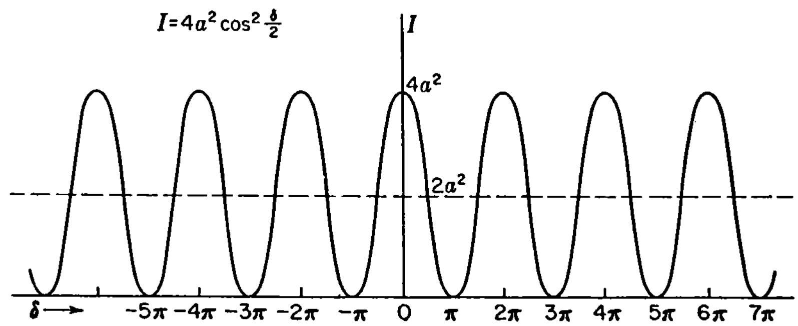


FIGURE 13H
Intensity distribution for the interference fringes from two waves of the same frequency.

the average intensity on the screen for n fringes, we note that the average value of the square of the cosine is $\frac{1}{2}$. This gives, by Eq. (13b), $I \approx 2a^2$, justifying the statement made above, and it shows that no violation of the law of conservation of energy is involved in the interference phenomenon.

13.5 FRESNEL'S BIPRISM*

Soon after the double-slit experiment was performed by Young, the objection was raised that the bright fringes he observed were probably due to some complicated modification of the light by the edges of the slits and not to true interference. Thus the wave theory of light was still questioned. Not many years passed, however, before Fresnel brought forward several new experiments in which the interference of two beams of light was proved in a manner not open to the above objection. One of these, the Fresnel biprism experiment, will be described in some detail.

A schematic diagram of the biprism experiment is shown in Fig. 13I. The thin double prism P refracts the light from the slit sources S into two overlapping beams ac and be. If screens M and N are placed as shown in the figure, interference fringes are observed only in the region bc. When the screen ae is replaced by a photographic plate, a picture like the upper one in Fig. 13J is obtained. The closely spaced fringes in the center of the photograph are due to interference, while the wide fringes at the edge of the pattern are due to diffraction. These wider bands are produced by the vertices of the two prisms, each of which acts as a straightedge, giving a pattern which will be discussed in detail in Chap. 18. When the screens M and N are removed from

* Augustin Fresnel (1788–1827). Most notable French contributor to the theory of light. Trained as an engineer, he became interested in light, and in 1814–1815 he rediscovered Young's principle of interference and extended it to complicated cases of diffraction. His mathematical investigations gave the wave theory a sound foundation.

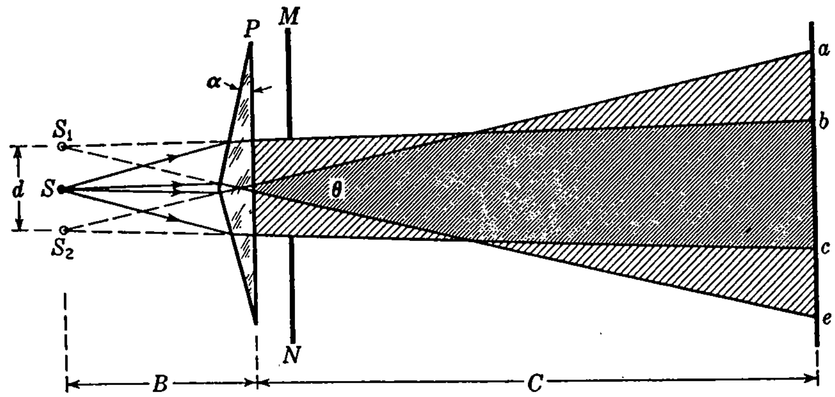


FIGURE 13I
Diagram of Fresnel's biprism experiment.

the light path, the two beams will overlap over the whole region *ae*. The lower photograph in Fig. 13J shows for this case the equally spaced interference fringes superimposed on the diffraction pattern of a wide aperture. (For the diffraction pattern above, without the interference fringes, see lowest figures in Fig. 18U.) With such an experiment Fresnel was able to produce interference without relying upon diffraction to bring the interfering beams together.

Just as in Young's double-slit experiment, the wavelength of light can be determined from measurements of the interference fringes produced by the biprism. Calling B and C the distances of the source and screen, respectively, from the prism P, d the distance between the virtual images S_1 and S_2, and Δx the distance between the successive fringes on the screen, the wavelength of the light is given from Eq. (13d) as

$$\lambda = \frac{\Delta x\, d}{B + C} \tag{13f}$$

Thus the virtual images S_1 and S_2 act like the two slit sources in Young's experiment.

In order to find d, the linear separation of the virtual sources, one can measure their angular separation θ on a spectrometer and assume, to sufficient accuracy, that $d = B\theta$. If the parallel light from the collimator covers both halves of the biprism, two images of the slit are produced and the angle θ between these is easily measured with the telescope. An even simpler measurement of this angle can be made by holding the prism close to one eye and viewing a round frosted light bulb. At a certain distance from the light the two images can be brought to the point where their inner edges just touch. The diameter of the bulb divided by the distance from the bulb to the prism then gives θ directly.

Fresnel biprisms are easily made from a small piece of glass, such as half a microscope slide, by beveling about $\frac{1}{8}$ to $\frac{1}{4}$ in. on one side. This requires very little grinding with ordinary abrasive materials and polishing with rouge, since the angle required is only about 1°.

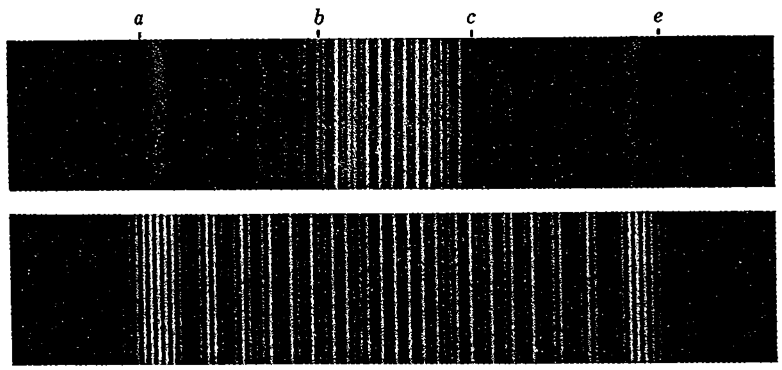


FIGURE 13J
Interference and diffraction fringes produced in the Fresnel biprism experiment.

13.6 OTHER APPARATUS DEPENDING ON DIVISION OF THE WAVE FRONT

Two beams can be brought together in other ways to produce interference. In the arrangement known as *Fresnel's mirrors*, light from a slit is reflected in two plane mirrors slightly inclined to each other. The mirrors produce two virtual images of the slit, as shown in Fig. 13K. They act in every respect like the images formed by the biprism, and interference fringes are observed in the region *bc*, where the reflected beams overlap. The symbols in this diagram correspond to those in Fig. 13I, and Eq. (13f) is again applicable. It will be noted that the angle 2θ subtended at the point of intersection M by the two sources is twice the angle between the mirrors.

The Fresnel double-mirror experiment is usually performed on an optical bench, with the light reflected from the mirrors at nearly grazing angles. Two pieces of ordinary plate glass about 2 in. square make a very good double mirror. One plate should have an adjusting screw for changing the angle θ and the other a screw for making the edges of the two mirrors parallel.

An even simpler device, shown in Fig. 13L, produces interference between the light reflected in one long mirror and the light coming directly from the source without reflection. In this arrangement, known as *Lloyd's mirror*, the quantitative relations are similar to those in the foregoing cases, with the slit and its virtual image constituting the double source. An important feature of the Lloyd's-mirror experiment lies in the fact that when the screen is placed in contact with the end of the mirror (in the position MN, Fig. 13L), the edge O of the reflecting surface comes at the center of a *dark* fringe, instead of a bright one as might be expected. This means that one of the two beams has undergone a phase change of π. Since the direct beam could not change phase, this experimental observation is interpreted to mean that the reflected light has changed phase at reflection. Two photographs of the Lloyd's-mirror fringes taken in this way are reproduced in Fig. 13M, one taken with visible light and the other with X rays.

If the light from source S_1 in Fig. 13L is allowed to enter the end of the glass

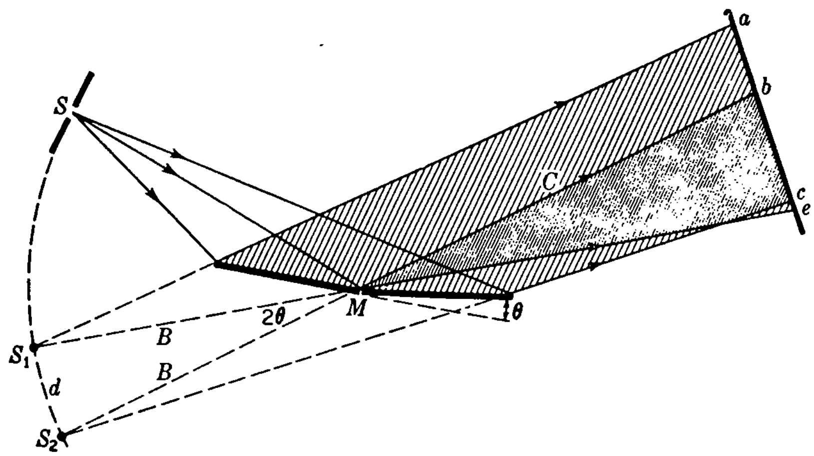


FIGURE 13K
Geometry of Fresnel's mirrors.

plate by moving the latter up, and to be internally reflected from the upper glass surface, fringes will again be observed in the interval OP, with a dark fringe at O. This shows that there is again a phase change of π at reflection. As will be shown in Chap. 25, this is not a contradiction of the discussion of phase change given in Sec. 14.1. In this instance the light is incident at an angle greater than the critical angle for total reflection.

Lloyd's mirror is readily set up for demonstration purposes as follows. A carbon arc, followed by a colored glass filter and a narrow slit, serves as a source. A strip of ordinary plate glass 1 to 2 in. wide and 1 ft or more long makes an excellent mirror. A magnifying glass focused on the far end of the mirror enables one to observe the fringes shown in Fig. 13M. Internal fringes can be observed by polishing the ends of

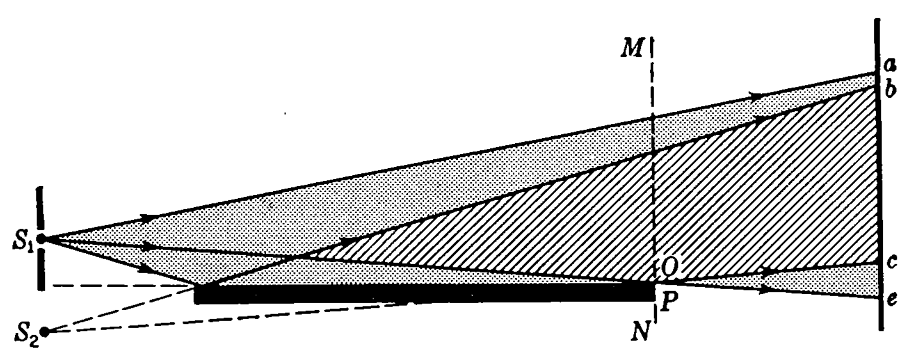
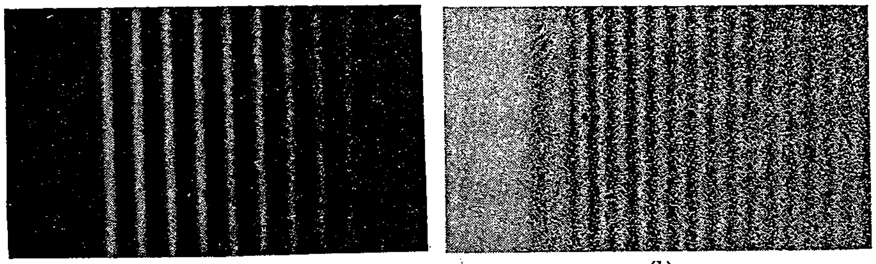


FIGURE 13L
Lloyd's mirror.



(a) (b)

FIGURE 13M
Interference fringes produced with Lloyd's mirror. (*a*) Taken with visible light, $\lambda = 4358$ Å. (*After White.*) (*b*) Taken with X rays, $\lambda = 8.33$ Å. (*After Kellstrom.*)

the mirror to allow the light to enter and leave the glass, and by roughening one of the glass faces with coarse emery.

Other ways exist* for dividing the wave front into two segments and subsequently recombining these at a small angle with each other. For example, one can cut a lens into two halves on a plane through the lens axis and separate the parts slightly, to form two closely adjacent real images of a slit. The images produced in this device, known as *Billet's split lens*, act like the two slits in Young's experiment. A single lens followed by a biplate (two plane-parallel plates at a slight angle) will accomplish the same result.

13.7 COHERENT SOURCES

It will be noticed that the various methods of demonstrating interference so far discussed have one important feature in common: the two interfering beams are always derived from the same source of light. We find by experiment that it is impossible to obtain interference fringes from two separate sources, such as two lamp filaments set side by side. This failure is caused by the fact that the light from any one source is not an infinite train of waves. On the contrary, there are sudden changes in phase occurring in very short intervals of time (of the order of 10^{-8} s). This point has already been mentioned in Secs. 11.1 and 12.6. Thus, although interference fringes may exist on the screen for such a short interval, they will shift their position each time there is a phase change, with the result that no fringes at all will be seen. In Young's experiment and in Fresnel's mirrors and biprism, the two sources S_1 and S_2 always have a point-to-point correspondence of phase, since they are both derived from the same source. If the phase of the light from a point in S_1 suddenly shifts, that of the light from the corresponding point in S_2 will shift simultaneously. The result is that the *difference* in phase between any pair of points in the two sources always remains constant, and so the interference fringes are stationary. It is a charac-

* Good descriptions will be found in T. Preston, "Theory of Light," 5th ed., chap. 7, The Macmillan Company, New York, 1928.

teristic of any interference experiment with light that the sources must have this point-to-point phase relation, and sources that have this relation are called *coherent sources.*

While special arrangements are necessary for producing coherent sources of light, the same is not true of *microwaves*, which are radio waves of a few centimeters wavelength. These are produced by an oscillator which emits a continuous wave, the phase of which remains constant over a time long compared with the duration of an observation. Two independent microwave sources of the same frequency are therefore coherent and can be used to demonstrate interference. Because of the convenient magnitude of their wavelength, microwaves are used to illustrate many common optical interference and diffraction effects.*

If in Young's experiment the source slit S (Fig. 13C) is made too wide or the angle between the rays which leave it too large, the double slit no longer represents two coherent sources and the interference fringes disappear. This subject will be discussed in more detail in Chap. 16.

13.8 DIVISION OF AMPLITUDE. MICHELSON† INTERFEROMETER

Interference apparatus may be conveniently divided into two main classes, those based on *division of wave front* and those based on *division of amplitude*. The previous examples all belong to the former class, in which the wave front is divided laterally into segments by mirrors or diaphragms. It is also possible to divide a wave by partial reflection, the two resulting wave fronts maintaining the original width but having reduced amplitudes. The Michelson interferometer is an important example of this second class. Here the two beams obtained by amplitude division are sent in quite different directions against plane mirrors, whence they are brought together again to form interference fringes. The arrangement is shown schematically in Fig. 13N. The main optical parts consist of two highly polished plane mirrors M_1 and M_2 and two plane-parallel plates of glass G_1 and G_2. Sometimes the rear side of the plate G_1 is lightly silvered (shown by the heavy line in the figure) so that the light coming from the source S is divided into (1) a reflected and (2) a transmitted beam of equal intensity. The light reflected normally from mirror M_1 passes through G_1 a third time and reaches the eye as shown. The light reflected from the mirror M_2 passes back through G_2 for the second time, is reflected from the surface of G_1 and into the

* The technique of such experiments is discussed by G. F. Hull, Jr., *Am. J. Phys.*, **17**:599 (1949).

† A. A. Michelson (1852–1931). American physicist of genius. He early became interested in the velocity of light and began experiments while an instructor in physics and chemistry at the Naval Academy, from which he graduated in 1873. It is related that the superintendent of the Academy asked young Michelson why he wasted his time on such useless experiments. Years later Michelson was awarded the Nobel prize (1907) for his work on light. Much of his work on the speed of light (Sec. 19.3) was done during 10 years spent at the Case Institute of Technology. During the latter part of his life he was professor of physics at the University of Chicago, where many of his famous experiments on the interference of light were done.

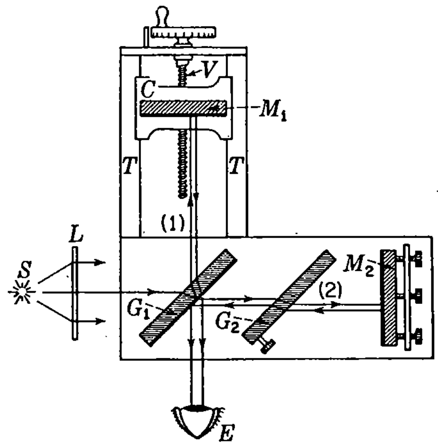


FIGURE 13N
Diagram of the Michelson interferometer.

eye. The purpose of the plate G_2, called the *compensating plate*, is to render the path *in glass* of the two rays equal. This is not essential for producing fringes in monochromatic light, but it is indispensable when white light is used (Sec. 13.11). The mirror M_1 is mounted on a carriage C and can be moved along the well-machined ways or tracks T. This slow and accurately controlled motion is accomplished by means of the screw V, which is calibrated to show the exact distance the mirror has been moved. To obtain fringes, the mirrors M_1 and M_2 are made exactly perpendicular to each other by means of screws shown on mirror M_2.

Even when the above adjustments have been made, fringes will not be seen unless two important requirements are fulfilled. First, the light must originate from an *extended* source. A point source or a slit source, as used in the methods previously described, will not produce the desired system of fringes in this case. The reason for this will appear when we consider the origin of the fringes. Second, the light must in general be *monochromatic*, or nearly so. Especially is this true if the distances of M_1 and M_2 from G_1 are appreciably different.

An extended source suitable for use with a Michelson interferometer may be obtained in any one of several ways. A sodium flame or a mercury arc, if large enough, may be used without the screen L shown in Fig. 13N. If the source is small, a ground-glass screen or a lens at L will extend the field of view. Looking at the mirror M_1 through the plate G_1, one then sees the whole mirror filled with light. In order to obtain the fringes, the next step is to measure the distances of M_1 and M_2 to the back surface of G_1 roughly with a millimeter scale and to move M_1 until they are the same to within a few millimeters. The mirror M_2 is now adjusted to be perpendicular to M_1 by observing the images of a common pin, or any sharp point, placed between the source and G_1. Two pairs of images will be seen, one coming from reflection at the front surface of G_1 and the other from reflection at its back surface. When the tilting screws on M_2 are turned until one pair of images falls exactly on the other, the interference fringes should appear. When they first appear, the fringes will not be clear unless the eye is focused on or near the back mirror M_1, so the observer should look constantly at this mirror while searching for the fringes.

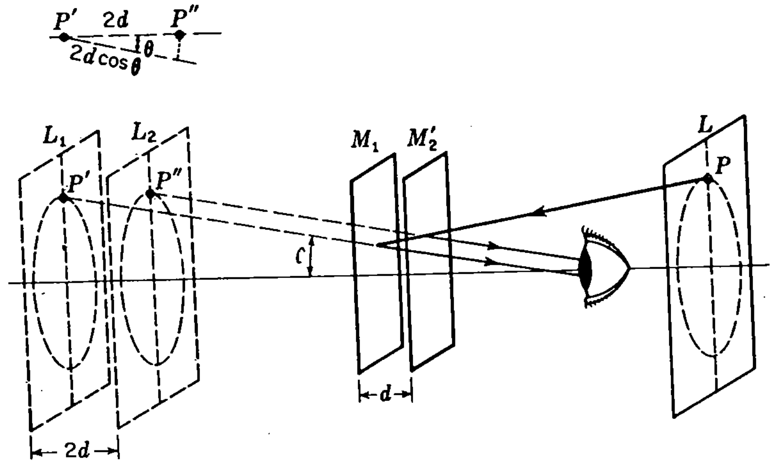


FIGURE 13O
Formation of circular fringes in the Michelson interferometer.

When they have been found, the adjusting screws should be turned in such a way as to continually increase the width of the fringes, and finally a set of concentric circular fringes will be obtained. M_2 is then exactly perpendicular to M_1 if the latter is at an angle of 45° with G_1.

13.9 CIRCULAR FRINGES

These are produced with monochromatic light when the mirrors are in exact adjustment and are the ones used in most kinds of measurement with the interferometer. Their origin can be understood by reference to the diagram of Fig. 13O. Here the real mirror M_2 has been replaced by its virtual image M_2' formed by reflection in G_1. M_2' is then parallel to M_1. Owing to the several reflections in the real interferometer, we may now think of the extended source as being at L, behind the observer, and as forming two virtual images L_1 and L_2 in M_1 and M_2'. These virtual sources are coherent in that the phases of corresponding points in the two are exactly the same at all instants. If d is the separation M_1M_2', the virtual sources will be separated by $2d$. When d is exactly an integral number of half wavelengths, i.e., the path difference $2d$ equal to an integral number of whole wavelengths, all rays of light reflected normal to the mirrors will be in phase. Rays of light reflected at an angle, however, will in general not be in phase. The path difference between the two rays coming to the eye from corresponding points P' and P'' is $2d \cos \theta$, as shown in the figure. The angle θ is necessarily the same for the two rays when M_1 is parallel to M_2' so that the rays are parallel. Hence when the eye is focused to receive parallel rays (a small telescope is more satisfactory here, especially for large values of d) the rays will reinforce each other to produce maxima for those angles θ satisfying the relation

$$2d \cos \theta = m\lambda \qquad (13g)$$

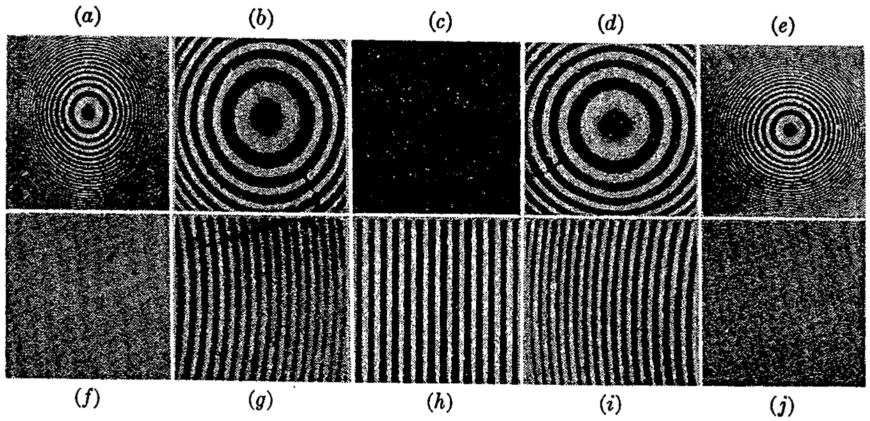


FIGURE 13P
Appearance of the various types of fringes observed in the Michelson interferometer. *Upper row*, circular fringes. *Lower row*, localized fringes. Path difference increases outward, in both directions, from the center.

Since for a given m, λ, and d the angle θ is constant, the maxima will lie in the form of circles about the foot of the perpendicular from the eye to the mirrors. By expanding the cosine, it can be shown from Eq. (13g) that the radii of the rings are proportional to the square roots of integers, as in the case of Newton's rings (Sec. 14.5). The intensity distribution across the rings follows Eq. (13b), in which the phase difference is given by

$$\delta = \frac{2\pi}{\lambda} 2d \cos \theta$$

Fringes of this kind, where parallel beams are brought to interference with a phase difference determined by the angle of inclination θ, are often referred to as *fringes of equal inclination.* In contrast to the type to be described in the next section, this type may remain visible over very large path differences. The eventual limitation on the path difference will be discussed in Sec. 13.12.

The upper part of Fig. 13P shows how the circular fringes look under different conditions. Starting with M_1 a few centimeters beyond M_2', the fringe system will have the general appearance shown in (*a*) with the rings very closely spaced. If M_1 is now moved slowly toward M_2' so that d is decreased, Eq. (13g) shows that a given ring, characterized by a given value of the order m, must decrease its radius because the product $2d \cos \theta$ must remain constant. The rings therefore shrink and vanish at the center, a ring disappearing each time $2d$ decreases by λ, or d by $\lambda/2$. This follows from the fact that at the center $\cos \theta = 1$, so that Eq. (13g) becomes

$$2d = m\lambda \qquad (13h)$$

To change m by unity, d must change by $\lambda/2$. Now as M_1 approaches M_2' the rings become more widely spaced, as indicated in Fig. 13P(*b*), until finally we reach a critical position where the central fringe has spread out to cover the whole field

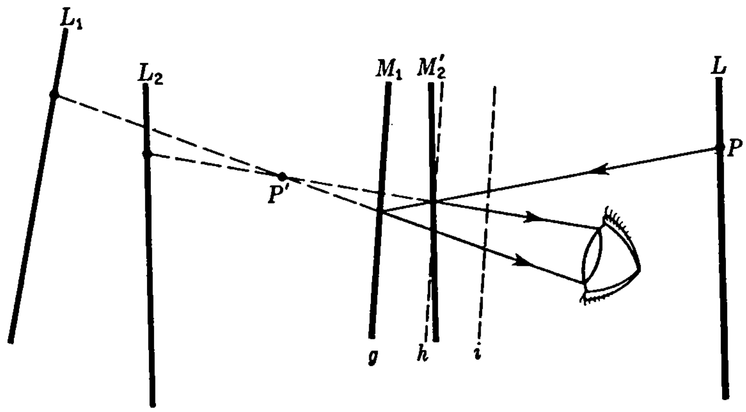


FIGURE 13Q
The formation of fringes with inclined mirrors in the Michelson interferometer.

of view, as shown in (*c*). This happens when M_1 and M_2' are exactly coincident, for it is clear that under these conditions the path difference is zero for all angles of incidence. If the mirror is moved still farther, it effectively passes through M_2', and new widely spaced fringes appear, growing out from the center. These will gradually become more closely spaced as the path difference increases, as indicated in (*d*) and (*e*) of the figure.

13.10 LOCALIZED FRINGES

If the mirrors M_2' and M_1 are not exactly parallel, fringes will still be seen with monochromatic light for path differences not exceeding a few millimeters. In this case the space between the mirrors is wedge-shaped, as indicated in Fig. 13Q. The two rays* reaching the eye from a point P on the source are now no longer parallel, but appear to diverge from a point P' near the mirrors. For various positions of P on the extended source, it can be shown† that the path difference between the two rays remains constant but that the distance of P' from the mirrors changes. If the angle between the mirrors is not too small, however, the latter distance is never great, and hence, in order to see these fringes clearly, the eye must be focused on or near the rear mirror M_1. The localized fringes are practically straight because the variation of the path difference across the field of view is now due primarily to the variation of the thickness of the "air film" between the mirrors. With a wedge-shaped film, the locus of points of equal thickness is a straight line parallel to the edge of the wedge. The

* When the term "ray" is used, here and elsewhere in discussing interference phenomena, it merely indicates the direction of the *perpendicular to a wave front* and is in no way to suggest an infinitesimally narrow pencil of light.

† R. W. Ditchburn, "Light," 2d ed., paperback, John Wiley and Sons, Inc., New York, 1963.

fringes are not exactly straight, however, if d has an appreciable value, because there is also some variation of the path difference with angle. They are in general curved and are always convex toward the thin edge of the wedge. Thus, with a certain value of d, we might observe fringes shaped like those of Fig. 13P(g). M_1 could then be in a position such as g of Fig. 13Q. If the separation of the mirrors is decreased, the fringes will move to the left across the field, a new fringe crossing the center each time d changes by $\lambda/2$. As we approach zero path difference, the fringes become straighter, until the point is reached where M_1 actually intersects M_2', when they are perfectly straight, as in (h). Beyond this point, they begin to curve in the opposite direction, as shown in (i). The blank fields (f) and (j) indicate that this type of fringe cannot be observed for large path differences. Because the principal variation of path difference results from a change of the thickness d, these fringes have been termed *fringes of equal thickness.*

13.11 WHITE-LIGHT FRINGES

If a source of white light is used, no fringes will be seen at all except for a path difference so small that it does not exceed a few wavelengths. In observing these fringes, the mirrors are tilted slightly as for localized fringes, and the position of M_1 is found where it intersects M_2'. With white light there will then be observed a central dark fringe, bordered on either side by 8 or 10 colored fringes. This position is often rather troublesome to find using white light only. It is best located approximately beforehand by finding the place where the localized fringes in monochromatic light become straight. Then a very *slow* motion of M_1 through this region, using white light, will bring these fringes into view.

The fact that only a few fringes are observed with white light is easily accounted for when we remember that such light contains all wavelengths between 400 and 750 nm. The fringes for a given color are more widely spaced the greater the wavelength. Thus the fringes in different colors will only coincide for $d = 0$, as indicated in Fig. 13R. The solid curve represents the intensity distribution in the fringes for green light, and the broken curve that for red light. Clearly, only the central fringe will be uncolored, and the fringes of different colors will begin to separate at once on either side, producing various impure colors which are not the saturated spectral colors. After 8 or 10 fringes, so many colors are present at a given point that the resultant color is essentially white. Interference is still occurring in this region, however, because a spectroscope will show a continuous spectrum with dark bands at those wavelengths for which the condition for destructive interference is fulfilled. White-light fringes are also observed in all the other methods of producing interference described above, if white light is substituted for monochromatic light. They are particularly important in the Michelson interferometer, where they may be used to locate the position of zero path difference, as we shall see in Sec. 13.13.

An excellent reproduction in color of these white-light fringes is given in one of Michelson's books.* The fringes in three different colors are also shown separately

* A. A. Michelson, "Light Waves and Their Uses," plate II, University of Chicago Press, Chicago, 1906.

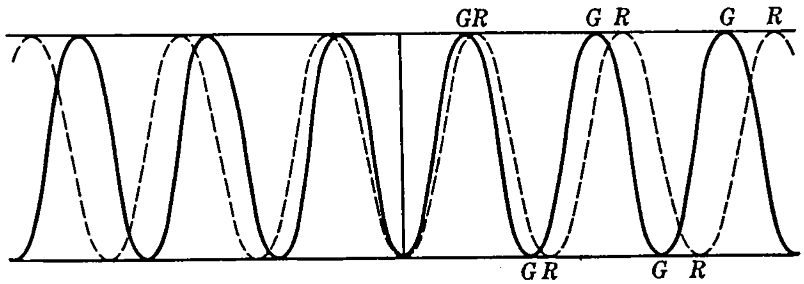


FIGURE 13R
The formation of white-light fringes with a dark fringe at the center.

and a study of these in connection with the white-light fringes is instructive as showing the origin of the various impure colors in the latter.

It was stated above that the central fringe in the white-light system, i.e., that corresponding to zero path difference, is black when observed in the Michelson interferometer. One would ordinarily expect this fringe to be white, since the two beams should be in phase with each other for any wavelength at this point, and in fact this is the case in the fringes formed with the other arrangements, such as the biprism. In the present case, however, it will be seen by referring to Fig. 13N that while ray 1 undergoes an internal reflection in the plate G_1, ray 2 undergoes an external reflection, with a consequent change of phase [see Eq. (14d)]. Hence the central fringe is black if the back surface of G_1 is unsilvered. If it is silvered, the conditions are different and the central fringe may be white.

13.12 VISIBILITY OF THE FRINGES

There are three principal types of measurement that can be made with the interferometer: (1) width and fine structure of spectrum lines, (2) lengths or displacements in terms of wavelengths of light, and (3) refractive indices. As explained in the preceding section, when a certain spread of wavelengths is present in the light source, the fringes become indistinct and eventually disappear as the path difference is increased. With white light they become invisible when d is only a few wavelengths, whereas the circular fringes obtained with the light of a single spectrum line can still be seen after the mirror has been moved several centimeters. Since no line is perfectly sharp, however, the different component wavelengths produce fringes of slightly different spacing, and hence there is a limit to the usable path difference even in this case. For the measurements of length to be described below, Michelson tested the lines from various sources and concluded that a certain red line in the spectrum of cadmium was the most satisfactory. He measured the *visibility*, defined as

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \qquad (13i)$$

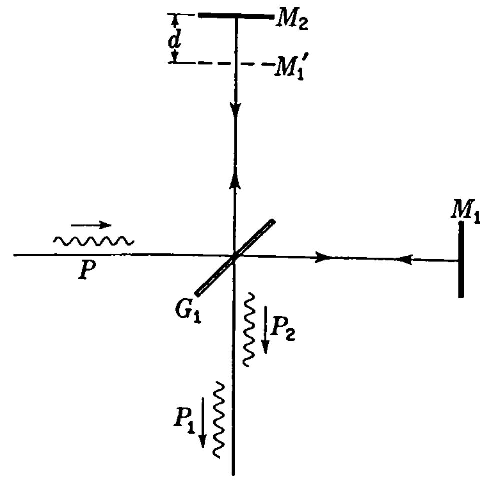


FIGURE 13S
Limiting path difference as determined by the length of wave packets.

where I_{max} and I_{min} are the intensities at the maxima and minima of the fringe pattern. The more slowly V decreases with increasing path difference, the sharper the line. With the red cadmium line, it dropped to 0.5 at a path difference of some 10 cm, or at $d = 5$ cm.

With certain lines, the visibility does not decrease uniformly but fluctuates with more or less regularity. This behavior indicates that the line has a fine structure, consisting of two or more lines very close together. Thus it is found that with sodium light the fringes become alternately sharp and diffuse, as the fringes from the two D lines get in and out of step. The number of fringes between two successive positions of maximum visibility is about 1000, indicating that the wavelengths of the components differ by approximately 1 part in 1000. In more complicated cases, the separation and intensities of the components could be determined by a Fourier analysis of the visibility curves.* Since this method of inferring the structure of lines has now been superseded by more direct methods, to be described in the following chapter, it will not be discussed in any detail here.

An alternative way of interpreting the eventual vanishing of interference at large path differences is instructive to consider at this point. In Sec. 12.6 it was indicated that a finite spread of wavelengths corresponds to wave packets of limited length, this length decreasing as the spread becomes greater. Thus, when the two beams in the interferometer traverse distances that differ by more than the length of the individual packets, these can no longer overlap and no interference is possible. The situation upon complete disappearance of the fringes is shown schematically in Fig. 13S. The original wave packet P has its amplitude divided at G_1 so that two similar packets are produced, P_1 traveling to M_1 and P_2 to M_2. When the beams are reunited, P_2 lags a distance $2d$ behind P_1. Evidently a measurement of this limiting path difference gives a direct determination of the length of the wave packets. This

* A. A. Michelson, "Studies in Optics," chap. 4, University of Chicago Press, Chicago, 1927.

interpretation of the cessation of interference seems at first sight to conflict with the one given above. A consideration of the principle of Fourier analysis shows, however, that mathematically the two are entirely equivalent and are merely alternative ways of representing the same phenomenon.

13.13 INTERFEROMETRIC MEASUREMENTS OF LENGTH

The principal advantage of Michelson's form of interferometer over the earlier methods of producing interference lies in the fact that the two beams are here widely separated and the path difference can be varied at will by moving the mirror or by introducing a refracting material in one of the beams. Corresponding to these two ways of changing the optical path, there are two other important applications of the interferometer. Accurate measurements of distance in terms of the wavelength of light will be discussed in this section, while interferometric determinations of refractive indices are described in Sec. 13.15.

When the mirror M_1 of Fig. 13N is moved slowly from one position to another, counting the number of fringes in monochromatic light which cross the center of the field of view will give a measure of the distance the mirror has moved in terms of λ, since by Eq. (13h) we have, for the position d_1 corresponding to the bright fringe of order m_1,

$$2d_1 = m_1\lambda$$

and for d_2, giving a bright fringe of order m_2,

$$2d_2 = m_2\lambda$$

Subtracting these two equations, we find

$$d_1 - d_2 = (m_1 - m_2)\frac{\lambda}{2} \qquad \text{(13j)}$$

Hence the distance moved equals the number of fringes counted, multiplied by a half wavelength. Of course, the distance measured need not correspond to an integral number of half wavelengths. Fractional parts of a whole fringe displacement can easily be estimated to one-tenth of a fringe, and, with care, to one-fiftieth. The latter figure then gives the distance to an accuracy of one-hundredth wavelength, or 5×10^{-7} cm for green light.

A small Michelson interferometer in which a microscope is attached to the moving carriage carrying M_1 is frequently used in the laboratory for measuring the wavelength of light. The microscope is focused on a fine glass scale, and the number of fringes, $m_1 - m_2$, crossing the mirror between two readings d_1 and d_2 on the scale gives λ, by Eq. (13j). The bending of a beam, or even of a brick wall, under pressure from the hand can be made visible and measured by attaching M_1 directly to the beam or wall.

The most important measurement made with the interferometer was the comparison of the standard meter in Paris with the wavelengths of intense red, green, and blue lines of cadmium by Michelson and Benoit. For reasons discussed in the last section, it would be impossible to count directly the number of fringes for a displacement of the movable mirror from one end of the standard meter to the other.

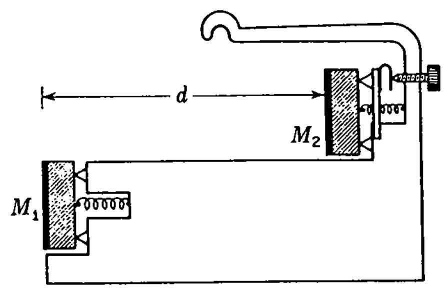


FIGURE 13T
One of the nine etalons used by Michelson in accurately comparing the wavelength of light with the standard meter.

Instead, nine intermediate standards (etalons) were used, of the form shown in Fig. 13T, each approximately twice the length of the other. The two shortest etalons were first mounted in an interferometer of special design (Fig. 13U), with a field of view covering the four mirrors, M_1, M_2, M_1', and M_2'. With the aid of the white light fringes the distances of M, M_1, and M_1' from the eye were made equal, as shown in the figure. Substituting the light of one of the cadmium lines for white light, M was then moved slowly from A to B, counting the number of fringes passing the cross hair. The count was continued until M reached the position B, which was exactly coplanar with M_2, as judged by the appearance of the white-light fringes in the upper mirror of the shorter etalon. The fraction of a cadmium fringe in excess of an integral number required to reach this position was determined, giving the distance M_1M_2 in terms of wavelengths. The shorter etalon was then moved through its own length, without counting fringes, until the white-light fringes reappeared in M_1. Finally M was moved to C, when the white-light fringes appeared in M_2' as well as in M_2. The additional displacement necessary to make M coplanar with M_2 was measured in terms of cadmium fringes, thus giving the exact number of wavelengths in the longer etalon. This was in turn compared with the length of a third etalon of approximately twice the length of the second, by the same process.

The length of the largest etalon was about 10.0 cm. This was finally compared with the prototype meter by alternately centering the white-light fringes in its upper and lower mirrors, each time the etalon was moved through its own length. Ten such steps brought a marker on the side of the etalon nearly into coincidence with the second fiducial mark on the meter, and the slight difference was evaluated by counting cadmium fringes. The 10 steps involve an accumulated error which does not enter in the intercomparison of the etalons, but nevertheless this was smaller than the uncertainty in setting on the end marks.

The final results were, for the three cadmium lines:

Red line	1 m = 1,553,163.5λ	or	λ = 6438.4722 Å
Green line	1 m = 1,966,249.7λ	or	λ = 5085.8240 Å
Blue line	1 m = 2,083,372.1λ	or	λ = 4799.9107 Å

Not only has the standard meter been determined in terms of what we now believe to be an invariable unit, the wavelength of light, but we have also obtained absolute determinations of the wavelength of three spectrum lines, the red line of

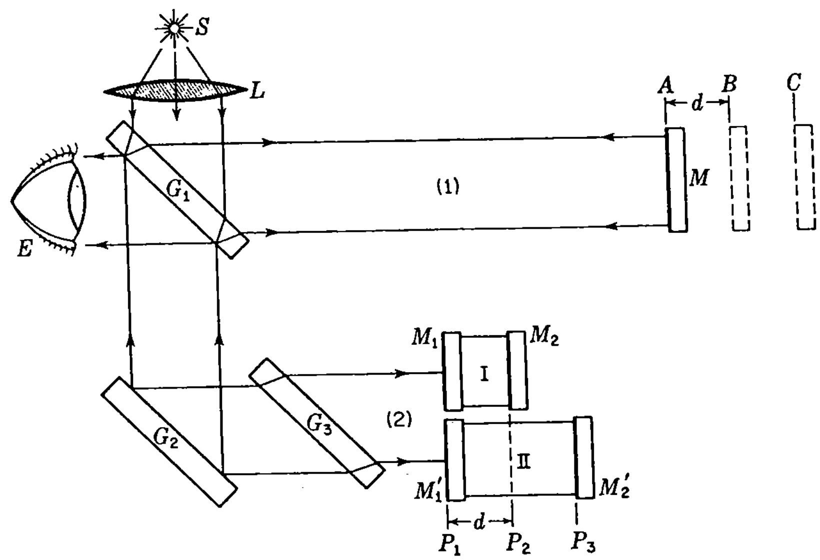


FIGURE 13U
Special Michelson interferometer used in accurately comparing the wavelength of light with the standard meter.

which is at present the primary standard in spectroscopy. More recent measurements on the orange line of the krypton spectrum have been made (see Sec. 14.11). It now is internationally agreed that in dry atmospheric air at 15°C and a pressure of 760 mmHg the orange line of krypton has a wavelength

$$\lambda_0 = 6057.80211 \text{ Å}$$

This is the wavelength the General Conference on Weights and Measures in Paris used in adopting on Oct. 14, 1960, as the international legal standard of length, the following definition of the standard meter:

$$1 \textit{ meter} = 1{,}650{,}763.73 \textit{ wavelengths (orange light of krypton)}$$

13.14 TWYMAN AND GREEN INTERFEROMETER

If a Michelson interferometer is illuminated with strictly parallel monochromatic light, produced by a point source at the principal focus of a well-corrected lens, it becomes a very powerful instrument for testing the perfection of optical parts such as prisms and lenses. The piece to be tested is placed in one of the light beams, and the mirror behind it is so chosen that the reflected waves, after traversing the test piece a second time, again become plane. These waves are then brought to interference with the plane waves from the other arm of the interferometer by another lens, at the focus of which the eye is placed. If the prism or lens is optically perfect, so that the returning waves are strictly plane, the field will appear uniformly illuminated. Any local variation of the optical path will, however, produce fringes in the corresponding part of the field, which are essentially the contour lines of the distorted wave front. Even though the surfaces of the test piece may be accurately made, the glass may contain regions that are slightly more or less dense. With the Twyman and Green interferometer these can be detected and corrected for by local polishing of the surface.*

13.15 INDEX OF REFRACTION BY INTERFERENCE METHODS

If a thickness t of a substance having an index of refraction n is introduced into the path of one of the interfering beams in the interferometer, the optical path in this beam is increased because of the fact that light travels more slowly in the substance and consequently has a shorter wavelength. The optical path [Eq. (1t)] is now nt through the medium, whereas it was practically t through the corresponding thickness of air ($n = 1$). Thus the increase in optical path due to insertion of the substance is $(n - 1)t$.† This will introduce $(n - 1)t/\lambda$ extra waves in the path of one beam; so if we call Δm the number of fringes by which the fringe system is displaced when the substance is placed in the beam, we have

$$(n - 1)t = (\Delta m)\lambda \qquad (13k)$$

In principle a measurement of Δm, t, and λ thus gives a determination of n.

In practice, the insertion of a plate of glass in one of the beams produces a discontinuous shift of the fringes so that the number Δm cannot be counted. With monochromatic fringes it is impossible to tell which fringe in the displaced set corresponds to one in the original set. With white light, the displacement in the fringes of different colors is very different because of the variation of n with wavelength, and the fringes disappear entirely. This illustrates the necessity of the compensating plate G_2 in Michelson's interferometer if white-light fringes are to be observed. If the plate of glass is very thin, these fringes may still be visible, and this affords a method of measuring n for very thin films. For thicker pieces, a practicable method is to use two plates of identical thickness, one in each beam, and to turn one gradually about a vertical axis, counting the number of monochromatic fringes for a given angle of rotation. This angle then corresponds to a certain known increase in effective thickness.

For the measurement of the index of refraction of gases, which can be introduced gradually into the light path by allowing the gas to flow into an evacuated tube, the interference method is the most practicable one. Several forms of refractometers have been devised especially for this purpose, of which we shall describe three, the Jamin, the Mach-Zehnder, and the Rayleigh refractometers.

Jamin's refractometer is shown schematically in Fig. 13V(a). Monochromatic

* For a more complete description of the use of this instrument, see F. Twyman, "Prism and Lens Making," 2d ed., chap. 12, Hilger and Watts, London, 1952.

† In the Michelson interferometer, where the beam traverses the substance twice in its back-and-forth path, t is *twice* the actual thickness.

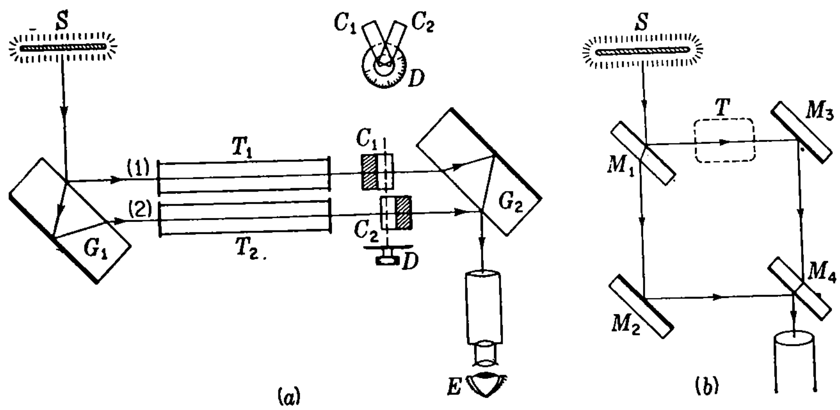


FIGURE 13V
(*a*) The Jamin and (*b*) the Mach-Zehnder interferometer.

light from a broad source S is broken into two parallel beams 1 and 2 by reflection at the two parallel faces of a thick plate of glass G_1. These two rays pass through to another identical plate of glass G_2 to recombine after reflection, forming interference fringes known as Brewster's fringes (see Sec. 14.11). If now the plates are parallel, the light paths will be identical. Suppose as an experiment we wish to measure the index of refraction of a certain gas at different temperatures and pressures. Two similar evacuated tubes T_1 and T_2 of equal length are placed in the two parallel beams. Gas is slowly admitted to tube T_2. If the number of fringes Δm crossing the field is counted while the gas reaches the desired pressure and temperature, the value of n can be found by applying Eq. (13k). It is found experimentally that at a given temperature the value $n - 1$ is directly proportional to the pressure. This is a special case of the *Lorenz-Lorentz* law*, according to which

$$\frac{n^2 - 1}{n^2 + 2} = (n - 1)\frac{n + 1}{n^2 + 2} = \text{const} \times \rho \qquad (13l)$$

Here ρ is the density of the gas. When n is very nearly unity, the factor $(n + 1)/(n^2 + 2)$ is nearly constant, as required by the above experimental observation.

The interferometer devised by Mach and Zehnder, and shown in Fig. 13V(*b*), has a similar arrangement of light paths, but they may be much farther apart. The role of the two glass blocks in the Jamin instrument is here taken by two pairs of mirrors, the pair M_1 and M_2 functioning like G_1, and the pair M_3 and M_4 like G_2. The first surface of M_1 and the second surface of M_4 are half-silvered. Although it is

* H. A. Lorentz (1853–1928). For many years professor of mathematical physics at the University of Leyden, Holland. Awarded the Nobel prize (1902) for his work on the relations between light, magnetism, and matter, he also contributed notably to other fields of physics. Gifted with a charming personality and kindly disposition, he traveled a great deal, and was widely known and liked. By a strange coincidence L. Lorenz of Copenhagen derived the above law from the elastic-solid theory only a few months before Lorentz obtained it from the electromagnetic theory.

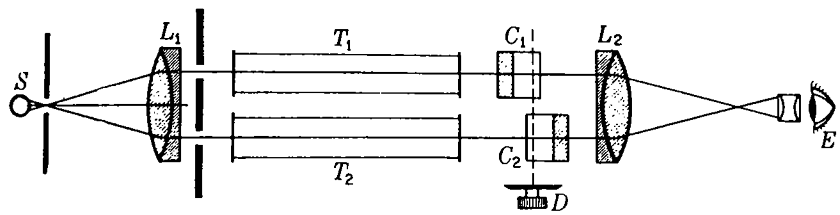


FIGURE 13W
Rayleigh's refractometer.

more difficult to adjust, the Mach-Zehnder interferometer is suitable only for studying slight changes of refractive index over a considerable area and is used, for example, in measuring the flow patterns in wind tunnels (see also Sec. 28.14). Contrary to the situation in the Michelson interferometer, the light traverses a region such as T in the figure in only one direction, a fact which simplifies the study of local changes of optical path in that region.

The purpose of the compensating plates C_1 and C_2 in Figs. 13V(*a*) and 13W is to speed up the measurement of refractive index. As the two plates, of equal thickness, are rotated together by the single knob attached to the dial D, one light path is shortened and the other lengthened. The device can therefore compensate for the path difference in the two tubes. The dial, if previously calibrated by counting fringes, can be made to read the index of refraction directly. The sensitivity of this device can be varied at will, a high sensitivity being obtained when the angle between the two plates is small and a low sensitivity when the angle is large.

In Rayleigh's* refractometer (Fig. 13W) monochromatic light from a linear source S is made parallel by a lens L_1 and split into two beams by a fairly wide double slit. After passing through two exactly similar tubes and the compensating plates, these are brought to interfere by the lens L_2. This form of refractometer is often used to measure slight differences in refractive index of liquids and solutions.

PROBLEMS

13.1 Young's experiment is performed with orange light from a krypton arc. If the fringes are measured with a micrometer eyepiece at a distance 100 cm from the double slit, it is found that 25 of them occupy a distance of 12.87 mm between centers. Find the distance between the centers of the two slits. *Ans.* 1.1297 mm

13.2 A double slit with a separation of 0.250 mm between centers is illuminated with green light from a cadmium-arc lamp. How far behind the slits must one go to measure the fringe separation and find it to be 0.80 mm between centers?

* Lord Rayleigh (third Baron) (1842–1919). Professor of physics at Cambridge University and the Royal Institution of Great Britain. Gifted with great mathematical ability and physical insight, he made important contributions to many fields of physics. His works on sound and on the scattering of light (Sec. 22.9) are the best known. He was a Nobel prize winner in 1904.

13.3 When a thin film of transparent plastic is placed over one of the slits in Young's double-slit experiment, the central bright fringe, of the white-light fringe system, is displaced by 4.50 fringes. The refractive index of the material is 1.480, and the effective wavelength of the light is 5500 Å. (*a*) By how much does the film increase the optical path? (*b*) What is the thickness of the film? (*c*) What would probably be observed if a piece of the material 1.0 mm thick were used? (*d*) Why?

13.4 Lloyd's-mirror experiment is readily demonstrated with microwaves, using as a reflector a sheet of metal lying flat on the table. If the source has a frequency of 12,000 MHz and is located 10.0 cm above the sheet-metal surface, find the height above the surface of the first two maxima 3.0 m from the source.

Ans. (*a*) 18.750 cm, (b) 56.25 cm

Note: A phase change of π occurs upon reflection; see Sec. 13.6.

13.5 A Fresnel biprism is to be constructed for use on an optical bench with the slit and the observing screen 180.0 cm apart. The biprism is to be 60.0 cm from the slit. Find the angle between the two refracting surfaces of the biprism if the glass has a refractive index $n = 1.520$, sodium yellow light is to be used, and the fringes are to be 1.0 mm apart.

13.6 A Fresnel biprism of index 1.7320 and with apex angles of 0.850° is used to form interference fringes. Find the fringe separation for red light of wavelength 6563 Å when the distance between the slit and the prism is 25.0 cm and that between the prism and the screen is 75.0 cm.

13.7 What must be the angle in degrees between the two Fresnel mirrors in order to produce sodium light fringes 1.0 mm apart if the slit is 40.0 cm from the mirror intersection and the screen is 150.0 cm from the slit? Assume $\lambda = 5.893 \times 10^{-5}$ cm.

Ans. 0.06331°

13.8 How far must the movable mirror of a Michelson interferometer be displaced for 2500 fringes of the red cadmium line to cross the center of the field of view?

13.9 If the mirror of a Michelson interferometer is moved 1.0 mm, how many fringes of the blue cadmium line will be counted crossing the field of view?

13.10 Find the angular radius of the tenth bright fringe in a Michelson interferometer when the central-path difference ($2d$) is (*a*) 1.50 mm and (*b*) 1.5 cm. Assume the orange light of a krypton arc is used and that the interferometer is adjusted in each case so that the first bright fringe forms a maximum at the center of the pattern.

Ans. (*a*) 4.885°, (*b*) 1.542°