X-Ray Project, Part I
PHYS 310

Introduction

The title of this project probably sounds to you as though this is just about a technique. And it is, to a certain extent. But really, this is several different projects, all of which can be executed on a really cool device that we have from LD Didactic that can not only produce x-ray sources but which can also use these x-ray sources to study (1) electronic structure of a range of atomic elements, (2) the crystal structure of many solids, and (3) the Compton Effect in which a single photon scatters of an elementary particle. (These are actually three separate projects – you will not have time to do all three of these.) But there is also a lot of interesting physics involved in understanding the production of x-rays. And x-ray devices are used *so* often in science and technology that it is important for y'all to have some familiarity in principles and techniques involving x-rays.

X-rays are commonly produced when beams of electrons slam into a material. Several scientists inadvertently produced x-rays throughout the 18th Century as a by-product of experiments that they were doing with electron beams and cathode-ray tubes, and some even noticed that they could be used for "seeing through" matter. They were first formally identified[1] by Wilhelm Röntgen[2] in 1895 who realized that he had evidence for a beam of something that was clearly not visible light. He dubbed them "X-rays" because he did not know what they actually were. It was later determined that x-rays are a form of electromagnetic radiation with a wavelength (frequency) that is smaller (larger) than that for visible light (or ultraviolet radiation). Wavelengths (frequencies) range from around 10 pm to 10 nm (3×10^{19} to 3×10^{16} Hz) for x-rays, corresponding to photon energies ($E = hf$) that range from ~ 100 eV to 100 keV. In large enough doses, x-rays can pose physical hazards since these energies exceed the binding energies in biologically-relevant molecules and can therefore cause mutations that can lead to cancer. On the other hand, when used judiciously, x-rays are incredibly useful for medical diagnostics since they can penetrate the body and can be used to form precise (and 3-D) maps of internal structures in the body, especially when used in conjunction with radiocontrast agents that effectively absorb x-rays.

Background and Theory

We will not go over the theory in detail in this write-up, because there are numerous sources, several of which we have in PDF form. The excerpt from "Elements of X-ray Diffraction," 2nd Edition (B. D. Cullity)[1] provides a good overview of the production and absorption of x-rays, including a discussion of *bremsstrahlung* (braking radiation) and of K_α and K_β emission spectra due to transitions in atomic electrons down into the lowest energy level (the *K-shell*). The excerpt from Ashcroft & Merman's *Solid State Physics*[3] covers the basics of microscopic crystalline structure and how that structure can be revealed by x-ray diffraction. And there are several "LD Physics Leaflets" that cover a range of both physics principles and the experimental techniques used to test and elucidate those principles using this device.

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[1] "Elements of X-ray Diffraction," 2nd Edition, B. D. Cullity, Addison-Wesley (1978).

[2] Röntgen was awarded the first ever Nobel Prize in physics for his "discovery" of x-rays.

[3] "Solid State Physics," Ashcroft & Mermin, Saunders (1976).

Safety

The x-ray device has several built-in safety features which pretty much guarantee that you will not be exposed to harmful levels of x-ray radiation. The second task below includes a discussion of safety features.

Tasks (you do not necessarily have to do the various tasks in this order) for Group #1

1. Read the handout titles "Properties of X-rays" (from *Elements of X-ray Diffraction*[1]).

2. Familiarize yourself with the x-ray device by following the instructions in the "X-Ray Production: General Overview of Apparatus" handout.

3. Read the handout "X-Ray Production: Determination of the Wavelengths of Characteristic Radiation from a Molybdenum and a Copper Anode" and follow the procedures to determine the wavelengths of the K_α and K_β emission lines for molybdenum and copper. After this step, you will switch back to the Mo anode x-ray tube for the rest of the project.

   For your presentations: First, you should explain how the device produces x-rays and how the spectrum is measured based on Bragg diffraction. You should be able to explain the shape of the spectrum including both the continuous spectrum due to *bremsstrahlung* and the discrete spectral peaks. **Questions:** (1) Can you explain the atomic transitions responsible for the K_α and K_β emission lines, respectively? (2) Why is it that the transition is from the *p* ($l = 1$) orbital of the L (n = 2) shell and not from the *s* ($l = 0$) orbital?

4. Determine the x-ray spectrum for a range of accelerator voltages, e.g., 35, 30, 25, 20 and 15 kV. Use the cut-off wavelength for each spectrum to determine Plank's constant *h* for each case, and then (using techniques discussed in our data analysis classes) do a weighted average of those results to determine your best estimate (with uncertainty) of Plank's constant. Also, note and be able to explain any qualitative differences in the x-ray spectrum for different accelerator voltages.

5. Do a complete test of Moseley's Law by measuring the spectrum of x-ray diffraction from a Mo anode (using the NaCl crystal) with the diffracted beam passing through various foil absorbers. For these experiments, you need only scan up to an angle of 11 or 12 degrees. You will first need the spectrum without any filter. Then do again with Zr, Mo, Ag and In filters installed in front of the detector. (Take care putting the filter on and removing the filter.)

   Once you have the five spectra, determine transmission curves ($R/R_{\text{no filter}}$) for each of the five filters and then identify the angle (with uncertainties) of the K-edge for each. From these results, determine wavelength λ_K for each and use these to test Moseley's Law:

$$\sqrt{\frac{1}{\lambda_K}} = \sqrt{R}(Z - \sigma_K)$$

where R is Rydberg's constant (accepted value 1.097 × 10^7 m^{-1}), Z is the atomic number of the filter material, and σ_K is the screening constant. Rydberg's constant can also be determined theoretically from

$$R = \frac{m_e e^4}{8\varepsilon_o^2 h^3 c}$$

where m_e and e are the mass and charge of the electron, ϵ_o is the permittivity of free space, h is Plank's constant and c is the speed of light.

You should be able to determine an experimental value for Rydberg's constant (with uncertainties) by plotting $\sqrt{\frac{1}{\lambda_K}}$ vs Z.

Questions for your presentation: when doing x-ray diagnostics for medical applications (e.g., CT scans), the patient is often injected with a barium or iodine *radiocontrast agent*. See if you can explain how these help improve the x-ray and CT scan images, based on the principle of K-edges and absorption of x-rays.

6. The K_α spectral line for Molybdenum is actually two, very closely-spaced emissions lines. This can be understood by considering spin-orbit coupling of the electrons in the L (n=2) shell. The angular momentum quantum number l can be either 0 or 1 (corresponding the magnitude of the orbital angular momentum of the electron) and, of course, the electron has a spin angular momentum quantum number ½. The orbital and spin angular momentums can combine to give a total angular momentum $\vec{J} = \vec{L} + \vec{S}$ with quantum number j = ½ or 3/2. And this total angular momentum results in a different magnetic moment μ for the orbiting electron in the L-shell. Interactions with the magnetic field produced by the nucleus results in a small difference in the energies of the j = ½ and 3/2 electrons in the L-shell, so there is a slight energy difference corresponding to the transitions of each of these electrons to the K-shell.

   To measure the splitting of the K_α emission line(s), you need to go to a higher-order spectrum to be able to see the two peaks. The 5th-order spectrum works, although (a) the rate is *much* lower in the 5th order, so you need to take data for a much longer time; and (b) even at this order, the two peaks are still overlapping with each other, so you will need to do nonlinear curve-fitting to identify the angles for each of the peaks. Fitting to the function: $R = A_0 e^{-(\vartheta - \vartheta 0)/b0} + A_1 e^{-(\vartheta - \vartheta 1)/b1} + c$ should work if the data is clean enough.

   For these experiments, you will want to do a run overnight. Re-zero the stage/detector with the NaCl crystal, and do a scan from 27 degrees to 42 degrees (if you have fifteen hours or so), collecting data every 0.1 degree and using a Δt of 400 seconds. Then do the nonlinear fit to determine the angles and wavelengths of the two emission lines (with uncertainties).

Questions for presentation: How do you think astronomers can use the splitting of the K_α spectral line to determine the magnetic field in a distant star or brown dwarf or whatever? (This is a technique that *is* used quite commonly in astronomy.)