1. With the minimum of fuss, show that the energy of an electron in an infinite square well (of length \( L \)) is given by \( E = \frac{n^2 \hbar^2}{8mL^2} \), where \( m \) = mass of electron, \( n = 1, 2, 3, \ldots \)

b) Calculate the first excited state of the electron when \( L = 1.27 \text{ Å} \).

4. Consider the case of an electron in a finite square well (1-dimensional, for the particular wavefunction shown). For the particular wavefunction shown:
   a) Identify the state (i.e., ground, excited state).
   b) Estimate the energy of the electron.

5. For the case of an electron in the hydrogen atom for which \( L = 1 \), determine:
   a) the magnitude of the angular momentum, \( L \).
   b) the possible orientations of \( L \) with the \( z \)-axis.
   c) Draw a vector diagram showing the possible orientations.

6. A hydrogen atom electron is in the state \( 1s^2 \). What are:
   a) Compute the electron's angular momentum.
   b) Compute the electron's angular momentum.
   c) Compute the possible values of \( L \).

Report on the answer to the question:

7. Show that the n-th energy level of an electron in the hydrogen atom has \( n^2 \) degeneracy.

8. It turns out that there are some constraints when describing the allowed transitions of hydrogen atom. In the case of transitions, some photons are emitted. Now, the photon has an intrinsic angular momentum which translates into a maximum component along any axis of \( z \text{-th} \).

a) To conserve angular momentum during a transition, show that the condition \( AD \neq +1 \).

b) Show that the constraint to the condition \( A \neq 0 \Rightarrow +1 \).

c) Show that all possible transitions for an electron in the 4s state. The answer is: 

\[ A \neq 0 \Rightarrow +1 \]
While the full wavefunction describing the electron in a hydrogen atom potential is given by
\[ \Psi(r, \theta, \phi, t) = \text{C} \exp[-\frac{\sqrt{2m}\cdot E}{\hbar^2}] \cdot f(\theta) g(\phi) \cdot \exp[-\frac{\sqrt{2m}\cdot E}{\hbar^2} t], \]
it is the probability density that matters

\[ P(r)\, dr = \text{prob. of finding the electron in a spherical shell between } r \& r+dr. \]

\[ \therefore P(r)\, dr = \Psi \cdot \Psi^* \, dv, \text{ where } dv = \text{small amount of spherical volume from } r \to r+dr. \]

a) Show \( dv = 4\pi r^2 \, dr \)

\& so \[ P(r)\, dr = 4\pi \Psi \cdot \Psi^* \cdot r^2 \, dr \]

Below are some radial probability density functions:

![Radial probability distribution figures]

Figure 8.11 The radial probability density function for several states of hydrogen-like atoms.

b) Write down (i.e. list) all trends that you observe.