Measurement of e/m:

J.J. Thomson's 2nd and more successful experiment, uses a cathode ray tube to provide a beam of particles. The particles then entered a region of perpendicular electric and magnetic fields, as shown below.

At first, the voltage (potential difference) controlling the electric field, $E$, was adjusted so that the particles passed by undeflected.

a) Show that the speed of the particles must necessarily satisfy

$$v = \frac{E}{B}.$$

The magnetic field was then turned off, causing the particles to deflect in the y-direction; their deflection was recorded on a distant screen.

b) There are two components contributing to the resultant deflection:

$y_1$ - the deflection caused while the particles are in the presence of the uniform electric field

$y_2$ - the deflection due to the y-component of the velocity as the particles leave the uniform E-field region and travel to the screen

Show that $y_1 = \frac{QE}{2m} (\frac{x_1}{u_x})^2$ and $y_2 = \frac{QE}{m} \frac{x_1 x_2}{u_x^2}$

where $u_x = \frac{E}{B}$.

c) Show that the overall deflection is

$$\frac{qe}{m} \left( \frac{B^2}{E} \right) \left( \frac{x_1^2}{a^2} + x_1 x_2 \right).$$
Mass Spectrometer:

Consider the arrangement shown to the right. Invented by Aston, a student of J.J. Thomson, this apparatus was used to obtain measurements of the charge-to-mass ratio for various ions.

Show that
\[ \frac{q}{m} = \frac{2\Delta V}{B^2 R^2} \]

Note that all the quantities on the right side of the equation are measurable.

Millikan Oil Drop Expt

This experiment is a refinement of the earlier experiment by J.J. Thomson, another student of J.J. Thomson, who used a small cloud of 'identical' water droplets drifting down at terminal velocity under the influence of gravity.

a) Read the account of the Millikan oil drop expt.

b) On drop #6, Millikan measured the following total charges at different times:
   - $25.41 \times 10^{-9}$ C
   - $17.47 \times 10^{-9}$ C
   - $12.70 \times 10^{-9}$ C
   - $20.64 \times 10^{-9}$ C
   - $19.06 \times 10^{-9}$ C
   - $14.29 \times 10^{-9}$ C

   Analyze these results to determine the fundamental unit of quantized charge.

c) Perform the analysis of drop #6 as described in the problem at the end of the reading.
Millikan's measurement of the charge on the electron is one of the few truly crucial experiments in physics and, at the same time, one whose simple directness serves as a standard against which to compare others. Figure 3-4 shows a sketch of Millikan's apparatus. With no electric field, the downward force on an oil drop is $mg$ and the upward force is $bv$. The equation of motion is

$$mg - bv = m \frac{dv}{dt}$$  \hspace{1cm} 3-10

where $b$ is given by Stokes' law:

$$b = 6\pi \eta a$$  \hspace{1cm} 3-11

and where $\eta$ is the coefficient of viscosity of the fluid (air) and $a$ is the radius of the drop. The terminal velocity of the falling drop $v_f$ is

Fig. 3-4 Schematic diagram of the Millikan oil-drop apparatus. The drops are sprayed from the atomizer and pick up a static charge, a few falling through the hole in the top plate. Their fall due to gravity and their rise due to the electric field between the capacitor plates can be observed with the telescope. From measurements of the rise and fall times, the electric charge on a drop can be calculated. The charge on a drop could be changed by exposure to x rays from a source (not shown) mounted opposite the light source.

(Continued)
\[ v_f = \frac{mg}{b} \quad 3-12 \]

(see Figure 3-5). When an electric field \( \mathcal{E} \) is applied, the upward motion of a charge \( q_n \) is given by

\[ q_n \mathcal{E} - mg - bv = m \frac{dv}{dt} \]

Thus the terminal velocity \( v_r \) of the drop rising in the presence of the electric field is

\[ v_r = \frac{q_n \mathcal{E} - mg}{b} \quad 3-13 \]

In this experiment, the terminal speeds were reached almost immediately, and the drops drifted a distance \( L \) upward or downward at a constant speed. Solving Equations 3-12 and 3-13 for \( q_n \), we have

\[ q_n = \frac{mg}{\mathcal{E}v_f} (v_f + v_r) = \frac{mgT_f}{\mathcal{E}} \left( \frac{1}{T_f} + \frac{1}{T_r} \right) \quad 3-14 \]

where \( T_f = L/v_f \) is the fall time and \( T_r = L/v_r \) is the rise time.

If any additional charge is picked up, the terminal velocity becomes \( v'_r \), which is related to the new charge \( q'_n \) by Equation 3-13:

\[ v'_r = \frac{q'_n \mathcal{E} - mg}{b} \]

The amount of charge gained is thus

\[ q'_n - q_n = \frac{mg}{\mathcal{E}v_f} (v'_r - v_r) \]

\[ = \frac{mgT_f}{\mathcal{E}} \left( \frac{1}{T'_r} - \frac{1}{T_r} \right) \quad 3-15 \]

The velocities \( v_f, v_r, \) and \( v'_r \) are determined by measuring the time taken to fall or rise the distance \( L \) between the capacitor plates.

If we write \( q_n = ne \) and \( q'_n - q_n = n'e \) where \( n' \) is the change in \( n \), Equations 3-14 and 3-15 can be written

\[ \frac{1}{n} \left( \frac{1}{T_f} + \frac{1}{T_r} \right) = \frac{\mathcal{E}e}{mgT_f} \quad 3-16 \]

and

\[ \frac{1}{n'} \left( \frac{1}{T'_r} - \frac{1}{T_r} \right) = \frac{\mathcal{E}e}{mgT_f} \quad 3-17 \]

(Continued)
To obtain the value of \( e \) from the measured fall and rise times, one needs to know the mass of the drop (or its radius, since the density is known). The radius is obtained from Stokes' law using Equation 3-12.

Notice that the right sides of Equations 3-16 and 3-17 are equal to the same constant, albeit an unknown one, since it contains the unknown \( e \). The technique, then, was to obtain a drop in the field of view and measure its fall time \( T_f \) (electric field off) and its rise time \( T_r \) (electric field on) for the unknown number of charges \( n \) on the drop. Then, for the same drop (hence, same mass \( m \)), \( n \) was changed by some unknown amount \( n' \) by exposing the drop to the x-ray source, thereby yielding a new value for \( n \); and \( T_f \) and \( T_r \) were measured. The number of charges on the drop was changed again and the fall and rise times recorded. This process was repeated over and over for as long as the drop could be held in view (or until the experimenter became tired), often for several hours at a time. The value of \( e \) was then determined by finding (basically by trial and error) the integer values of \( n \) and \( n' \) that made the left sides of Equations 3-16 and 3-17 equal to the same constant for all measurements using a given drop.

Millikan did experiments like these with thousands of drops, some of nonconducting oil, some of semiconductors like glycerine, and some of conductors like mercury. In no case was a charge found that was a fractional part of \( e \). This process, which you will have the opportunity to work with in solving the problem below using actual data from Millikan's sixth drop, yielded a value of \( e \) of \( 1.591 \times 10^{-19} \text{C} \). This value was accepted for about 20 years, until it was discovered that x-ray diffraction measurements of \( N_A \) gave values of \( e \) that differed from Millikan's by about 0.4 percent. The discrepancy was traced to the value of the coefficient of viscosity \( \eta \) used by Millikan, which was too low. Improved measurements of \( \eta \) gave a value about 0.5 percent higher, thus changing the value of \( e \) resulting from the oil-drop experiment to \( 1.601 \times 10^{-19} \text{C} \), in good agreement with the x-ray diffraction data. The modern "best" values of \( e \) and other physical constants are published periodically by the International Council of Scientific Unions. The currently accepted value of the electron charge is

\[
e = 1.60217733 \times 10^{-19} \text{C}
\]

with an uncertainty of 0.30 parts per million. Our needs in this book are rarely as precise as this, so we will typically use \( e = 1.602 \times 10^{-19} \text{C} \). Note that, while we have been able to measure the value of the quantized electric charge, there is no hint in any of the above as to why it has this value, nor do we know the answer to that question now.

Hardly a matter of only historical interest, Millikan's technique is currently being used in an ongoing search for elementary particles with fractional electric charge by M. Perl and co-workers.

**Problem**

The accompanying table shows a portion of the data collected by Millikan for drop number 6 in the oil-drop experiment. (a) Find the terminal fall velocity \( v_f \) from the table using the mean fall time and the fall distance (10.21 mm). (b) Use the density of oil \( \rho = 0.943 \text{g/cm}^3 = 943 \text{kg/m}^3 \), the viscosity of air \( \eta = 1.824 \times 10^{-5} \text{N}\cdot\text{s/m}^2 \), and \( g = 9.81 \text{m/s}^2 \) to calculate the radius \( a \) of the oil drop from Stokes' law as expressed in Equation 3-12. (c) The correct "trial value" of \( n \) is filled in in column 7. Determine the remaining correct values for \( n \) and \( n' \), in columns 4 and 7, respectively. (d) Compute \( e \) from the data in the table.

(Continued)
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<th>$T_r$</th>
<th>$T_r$</th>
<th>$1/T_r - 1/T_r$</th>
<th>$n'$</th>
<th>$(1/n')(1/T_r - 1/T_r)$</th>
<th>$1/T_r + 1/T_r$</th>
<th>$n$</th>
<th>$(1/n)(1/T_r - 1/T_r)$</th>
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