Problem 1
Consider a mass $m$ attached to the end of a pendulum of length $\ell$. The pendulum is suspended vertically from a ceiling, displaced slightly from its equilibrium position, and released from rest.

a) Using Newton’s second law, derive the equation of motion and show that the mass exhibits simple harmonic motion. Determine the natural frequency of the system.

b) Using energy conservation, show that the mass exhibits simple harmonic motion. Determine the natural frequency of the system.

c) List all approximations used in a) and b) above.

Problem 2 Oscillating floating object

This problem is an analysis of the floating object discussed in French, Ch 3, pages 49-51.

A metal cylinder of mass $m$, length $L$ and cross sectional area $A$ is suspended end up in a liquid of density $\rho$ as illustrated in Fig. 1.

The pressure at depth $d$ below the surface of the liquid is

$$P = P_{\text{surface}} + \rho gd$$

where $P_{\text{surface}}$ is the air pressure at the surface. The cylinder is displaced so that it is never completely submerged.
a) By considering all the forces acting on the cylinder and assuming that the air pressure at the top of the cylinder is the same as that at the surface of the liquid, show that the equilibrium position of the top of the cylinder is

\[ y_{eq} = L - \frac{m}{\rho A} \]  

(1)

b) Now suppose that the cylinder is displaced. By applying Newton’s second law and using Eq. 1, show that

\[ m \frac{d^2 y}{dt^2} = \rho g A (y_{eq} - y). \]

c) The displacement from equilibrium is \( y_d \equiv y - y_{eq} \). Show that

\[ m \frac{d^2 y_d}{dt^2} = -\rho g A y_d. \]

Use this to determine an expression for the angular frequency of oscillation of the cylinder.

d) Suppose that a log of mass 500 kg and diameter 0.50 m floats in this fashion in water. Determine the period of oscillation of the log if it manages to stay oriented end up.

e) Describe how you could use the oscillations of a cylinder of known mass and cross-sectional area to determine the density of an unknown liquid.

**Problem 3** Solutions to the simple harmonic oscillator equation of motion

Consider a simple harmonic oscillator satisfying

\[ \frac{d^2 x}{dt^2} = -\omega^2 x \]

where \( \omega \neq 0 \)

a) Show, by substitution that \( x(t) = Ct \), where \( C \neq 0 \) is a constant is not a solution to the equation of motion.

b) Show, by substitution that \( x(t) = Ct \cos(\omega t) \) where \( C \neq 0 \) is a constant is not a solution to the equation of motion.

c) Show explicitly that \( x(t) = 7 \cos(3t + 3) \), with \( x(t) \) in meters and \( t \) in seconds satisfies this equation.

d) Find the angular frequency, frequency, and period of the oscillator in the previous part.

**Problem 4**

Read through the complex exponential exercises described in questions 1,2,3,4,5, &7 of chapter 1 of French and summarise the results. Be sure you are familiar and versatile with these results.
Problem 5
Professor Krishna Chowdary invented a new playground ride: the vertical spring swing. The swing spring consists of a long spring with spring constant, $k = 250 \text{ N/m}$, hung vertically from a support, with a platform on the bottom on which a child can stand.

You are observing two children playing on the Chowdary spring swing. One child, of mass 25 kg, is standing motionless on the platform. Her friend pushes down on the platform with all his might, moving the platform down 1.0 m before letting go. The girl on the platform begin to oscillate up and down. You notice the amplitude of her oscillation is decreasing, and that after 30 sec, her amplitude is only about 0.1 m.

a) Determine an estimate for the damping constant $\gamma$ for the system.

b) You the notice that the girl is able to increase her amplitude, returning to her original amplitude and exceeding it! She does this without the help of her friend, simply by moving her body up and down. Explain how she is able to do this, and determine how many times a minute she must move up and down to get the most out of this effect.

Exercises for the Interested:
French: Chapter 1, question 9.