Frequency Response

The frequency response $H(w)$ of a system is a complex number for each frequency $w$ (rad/sec).

Polar form $\Rightarrow H(w) = |H(w)| \angle H(w)$

What does $H(w)$ tell us about a system?

Consider applying an input signal to the system that is a cosine with frequency $w_0$:

```
<table>
<thead>
<tr>
<th>System</th>
</tr>
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<tbody>
<tr>
<td>$H(w)$</td>
</tr>
</tbody>
</table>
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Time Signal: $A \cos [w_0 t] \quad \cdots \quad \cos [w_0 t + \_ \_ \_]$

Phasor: $A \angle 0^\circ$

$\Rightarrow H(w)$ describes the "amplitude gain" and the "phase shift" of the system at each frequency $w$. 
Example: Consider the RC circuit that we demonstrated in class.

\[
\hat{V}_{\text{out}} = \frac{\hat{V}_{\text{in}}}{R + \frac{1}{j \omega C}} \quad \hat{V}_{\text{in}} = \left(\frac{1}{1 + j \omega RC}\right) \hat{V}_{\text{in}}
\]

\[H(\omega) = \frac{\hat{V}_{\text{out}}}{\hat{V}_{\text{in}}} = \frac{1}{1 + j \omega RC}\]

For the values we considered in class:

- \( R = 1.1 \ \text{k}\Omega \)
- \( C = 0.22 \ \mu\text{F} \)
- \( \omega = 2\pi \times 658 \ \text{rad/sec} \)

\[H(2\pi \times 658) = \frac{1}{1 + j 1} = \frac{1}{\sqrt{2}} \angle -90^\circ \]

\[= 0.707 \angle -90^\circ \]

\[V_{\text{in}}(t) = 0.1 \cos(2\pi 658t) \quad \hat{V}_{\text{in}} = 0.1 \angle 0^\circ \]

\[V_{\text{out}}(t) = 0.0707 \cos(2\pi 658t -950) \quad \hat{V}_{\text{out}} = 0.0707 \angle -950^\circ \]