Laboratory 3
Convolution: Computations and Applications

The main objective in lab this week is to practice computing the convolution integral. We will also learn about the impulse response of continuous–time systems, where you will determine the impulse response of an RC circuit two ways, analytically and experimentally. Please work in pairs on these lab exercises.

1. Do you have any questions about Lab 2 or the Matlab programs that you used?

2. Consider a series RC circuit driven by a voltage source, with the output voltage measured across the capacitor. We will use R = 10 ohms and C = 0.1 microfarads.
   - Analyze this circuit and derive the expression for the impulse response, h(t). In your analysis, consider applying a rectangular pulse that gets briefer and briefer while maintaining unit area.
   - Devise a procedure to experimentally measure the impulse response of the circuit. (Hint: Input a square wave, and use the DC offset and duty cycle features of the function generator to make the pulses brief. Try using pulses with area = 0.001 volt–seconds.)
   - Use convolution to predict the response of this circuit to a 2 volt step function, f(t) = 2*u(t). Then apply a 2 volt square wave and compare the response with your prediction.

3. Find the impulse response h(t) of an ideal integrator, i.e. a system whose output y(t) is the integral of the input x(t), as in y(t) = \int_{-\infty}^{t} x(\lambda) d\lambda.

4. Convolve a unit step function with itself: y(t) = u(t) * u(t), and sketch y(t).

5. Perform the four convolution exercises on the attached sheet. Sketch the results and show them to a lab instructor for verification.

6. Consider the signals s_1(t), s_2(t) and filters with impulse response h_1(t), h_2(t) as shown below. Compute the output of each filter due to each input. That is, compute the four convolutions y_{11}(t) = s_1(t) * h_1(t), y_{12}(t) = s_1(t) * h_2(t), y_{21}(t) = s_2(t) * h_1(t), y_{22}(t) = s_2(t) * h_2(t).

These signals and filters are commonly used in digital communication systems that transmit bits (0s and 1s) from one place to another.
7. An application of convolution:

The MATLAB script mus.m passes digitized music through discrete–time systems with various impulse responses and then plays the resulting music. Simulations of this type are used to understand how an audio speaker or a listening room with a certain impulse response will affect the music that is heard in the room. The impulse response can be measured easily in practice in order to obtain a model for a listening room.

Run the Matlab script mus.m. You will also need to download the file slove.au and save it in the same directory as mus.m. The original music will be played, followed by the music convolved with \( g(t) = (2 \pi 300) \exp(-2 \pi 300 t) \), and then the music convolved with a different function \( h(t) \). The impulse responses \( g(t) \) and \( h(t) \) will be plotted on your screen. The program takes a while to run, so be patient!

You can listen to the results here, without running the MATLAB program:

- Original music
- Result of filtering with \( g(t) \)
- Result of filtering with \( h(t) \)

Please write brief answers to the following questions. You don’t have to submit any plots.

- What effect does convolution with \( g(t) \) have on the music, i.e. what is different about the music after the processing? Can you explain this effect from the shape of \( g(t) \)? (Hint: Does \( g(t) \) resemble the impulse response of an RC circuit? What type of filter does \( g(t) \) describe, and what is the cutoff frequency?)
- What effect does convolution with \( h(t) \) have on the music? Can you relate this effect to the shape of \( h(t) \)? What physical mechanism might give rise to an effect like this in a concert hall?

Lab Reports: Each pair of students is required to submit a report explaining your answers to items 2 through 7. This "report" can be hand-written, and is actually more like a homework assignment. The objective is for you to practice with convolution computations, show all of the steps in your solutions, and hear the effects of convolution on sound signals.

All reports are due on Friday, October 8 at 9 AM.

Thank you, and have fun!
Convolution exercises:

1. \[ \begin{array}{c}
\text{Input} \\
\frac{3}{2} \quad \frac{5}{2}
\end{array} \quad \ast \quad \begin{array}{c}
\text{Impulse Response} \\
\frac{5}{2} \quad \frac{3}{2}
\end{array} \]

2. \[ \begin{array}{c}
\text{Input} \\
0
\end{array} \quad \ast \quad \begin{array}{c}
\text{Impulse Response} \\
2u(t-1)
\end{array} \]

3. \[ \begin{array}{c}
\text{Input} \\
0 \quad 2
\end{array} \quad \ast \quad \begin{array}{c}
\text{Impulse Response} \\
3e^{-4t}u(t)
\end{array} \]

4. \[ \begin{array}{c}
\text{Input} \\
0 \quad 2 \quad 3
\end{array} \quad \ast \quad \begin{array}{c}
\text{Impulse Response} \\
\delta(t-3)
\end{array} \]