Rules: You may use the Lathi textbook, class notes, a calculator, and Matlab to complete this exam. However, you must work alone, and you are not permitted to discuss the exam with anyone.

Due Date: The exam is due by 4 PM on Wednesday, March 3. Professor Kozick will be in Dana 129 from 3-5 PM on March 3, so you can submit your exam in Dana 129 at that time. If you would like to submit your exam earlier, please give it to Debbie McAllister in the EE department office (Dana 346).

Show your work and explain your reasoning for full credit. Feel free to attach additional pages containing your answers. Good luck and have fun!

Problem 1. (10 points) Analog versus digital communication

Please provide brief answers to the following questions. Write the answers in your own words, using simple language that would be understandable to friend or family member who is not an engineer. Attach a separate page with your answers.

Briefly explain the difference between analog and digital communication systems.
What is an advantage of analog communication compared with digital communication?
What is an advantage of digital communication compared with analog communication?

Problem 2. (10 points) Channel capacity

Recall Shannon’s channel capacity formula,

\[ C = B \log_2 (1 + \text{SNR}) \text{ bits/sec}, \]

where \( B \) is the channel bandwidth (in hertz) and SNR is the signal-to-noise ratio. The SNR is the ratio of average signal power to average noise power.

Suppose that in a certain communication system, you have the option to either increase the channel bandwidth by a factor of 2 or increase the average signal power by a factor of 2. Which will have the greater impact on channel capacity, assuming that \( \text{SNR} \gg 1 \)?

Please explain your reasoning. Also, provide some numerical examples that illustrate your conclusion (i.e., evaluate the capacity for some particular values of \( B \) and SNR).
Problem 3. (40 points) LTI system analysis

Consider a linear, time-invariant system with frequency response $H(f)$ shown below. Please note that the axis on this frequency response is labeled in units of hertz, so you should sketch the equivalent frequency response $H(\omega)$ with $\omega$ in rad/sec.

a. Express $H(\omega)$ in terms of the rect(·) function.

b. What is the impulse response $h(t)$ of this system?
   Find the analytical expression for $h(t)$ and sketch $h(t)$. Include numerical values and units on the axes in your sketch of $h(t)$.

c. Consider a signal $x(t) = 0.2\delta(t) + 0.1\delta(t - 0.02)$, which is plotted above. Suppose that $x(t)$ is applied at the input to this system that has frequency response $H(\omega)$. What is the output $y(t)$ from the system if the input is $x(t)$? Sketch $y(t)$ and explain your reasoning.

d. Now suppose that the following input signal is applied to this same system:

$$g(t) = 19 + 30 \cos (2\pi 35t) + 75 \cos \left(2\pi 70t + \frac{\pi}{3}\right).$$

What is the steady-state output from the system if the input is $g(t)$?
Problem 4. (15 points) Fourier transform

Consider the design of a simple modem that will transmit bits over a dial-up telephone line. We will model the telephone channel as an ideal band-pass filter that passes frequencies in the range from 500 to 3,500 Hz. Bits (0’s and 1’s) will be transmitted by the polarity of a pulse. If the pulse is denoted by \( g(t) \), then a 0 is sent as \(-g(t)\) and a 1 is sent as \(+g(t)\).

One possible shape for the pulse in this system is
\[
g_1(t) = \text{rect} \left( \frac{t}{0.002} \right) \cdot \cos \left( 2\pi 2000t \right),
\]
and another possible shape for the pulse is
\[
g_2(t) = \Delta \left( \frac{t}{0.004} \right) \cdot \cos \left( 2\pi 2000t \right),
\]
where \(\Delta(\cdot)\) is the unit triangle function (see page 79 in Lathi).

Discuss the advantages and disadvantages of each pulse shape for this system. You may want to think about the following items.

- Sketch the time functions \( g_1(t), g_2(t) \) and the corresponding Fourier transforms \( G_1(\omega), G_2(\omega) \).
- How do the pulse durations compare in the time domain? How will this affect the bit rate?
- How do the frequency spectra, \( G_1(\omega) \) and \( G_2(\omega) \), compare? Which signal spectrum is a better “fit” to the channel, which only passes frequencies from 500 to 3,500 Hz? Which pulse, \( g_1(t) \) or \( g_2(t) \), will suffer less distortion after passing through the channel?
Problem 5. (25 points) Signal approximation

We will use the basis signals $x(t)$ and $y(t)$ shown below to approximate another signal, $g(t)$.

![Graph showing basis signals $x(t)$ and $y(t)$]

a. Find the energy of $x(t)$ and $y(t)$, $E_x$ and $E_y$.

b. Find the inner product (dot product) between $x(t)$ and $y(t)$. Are they orthogonal?

c. Consider a signal that is a half-cycle of a sine wave,

$$g(t) = 3 \sin(\pi t), \quad 0 \leq t \leq 1,$$

where $g(t) = 0$ for $t < 0$ and $t > 1$. Make a sketch of $g(t)$.

d. Find the approximation for $g(t)$ of the form

$$\hat{g}(t) = c_1 x(t) + c_2 y(t)$$

that has minimum error energy. That is, find the coefficients $c_1$ and $c_2$ that minimize

$$\int_0^1 [g(t) - \hat{g}(t)]^2 \, dt.$$

Show your work and explain your reasoning.

e. Make a sketch showing $g(t)$ and the approximation $\hat{g}(t)$ on the same graph.