Class example on independent random variables

Problem Solution: Yates and Goodman, 3.7.1

Problem 3.7.1

Flip a fair coin 100 times and let $X$ be the number of heads in the first 75 flips and $Y$ be the number of heads in the last 25 flips. We know that $X$ and $Y$ are independent and can find their PMFs easily.

$$P_X(x) = \begin{cases} \binom{75}{x} \left(\frac{1}{2}\right)^{75} & x = 0, 1, \ldots, 75 \\ 0 & \text{otherwise} \end{cases} \quad P_Y(y) = \begin{cases} \binom{25}{y} \left(\frac{1}{2}\right)^{25} & y = 0, 1, \ldots, 25 \\ 0 & \text{otherwise} \end{cases}$$

The joint PMF of $X$ and $N$ can be expressed as the product of the marginal PMFs because we know that $X$ and $Y$ are independent.

$$P_{X,Y}(x,y) = \begin{cases} \binom{75}{x} \binom{25}{y} \left(\frac{1}{2}\right)^{100} & x = 0, 1, \ldots, 75 \quad y = 0, 1, \ldots, 25 \\ 0 & \text{otherwise} \end{cases}$$