Problem E

Derive the appropriate Euler-Lagrange equation for the case where the integral $S$ has the form

$$S = \int_{x_1}^{x_2} f\left(y(x), y'(x), y''(x), x\right) \, dx$$

that is, where the integrand can depend on $y''$ as well as $y$ and $y'$. The values of $y$ and $y'$ are specified at the end points $x_1$ and $x_2$. This means the variation of the path, $\eta(x)$, must satisfy all of

$$\eta(x_1) = \eta(x_2) = 0, \quad \text{and} \quad \eta'(x_1) = \eta'(x_2) = 0.$$