Homework Assignment #2

due: Wednesday, September 20, 1pm

1. Vector Space *(Griffiths problem 3.2)*
Consider the collection of all polynomials (with complex coefficients) of degree < N in x.
(a) Does this set constitute a vector space (with the polynomials as “vectors”)? If so, suggest a convenient basis, and give the dimension of the space. If not, which of the defining properties does it lack?
(b) What if we require that the polynomials be even functions?
(c) What if we require that the leading coefficient (i.e. the number multiplying $x^{N-1}$) be 1?
(d) What if we require that the polynomials have the value 0 at $x = 1$ ?
(e) What if we require that the polynomials have the value 1 at $x = 0$ ?

2. Linear Transformation
Consider the vector space of polynomials of order two, i.e. $f(x) = a_0 + a_1 x + a_2 x^2$. Is the transformation $T$ defined as $T(f) = (a_1 + 1)x$ a linear transformation?

3. Projection Operator
Townsend problem 2.2 (page 61)  (2P)

4. Column Vectors
Townsend problem 2.4 (page 61)
*Hint:* Use that $|+x\rangle = \frac{1}{\sqrt{2}} |+z\rangle + \frac{1}{\sqrt{2}} |-z\rangle$, $|-x\rangle = \frac{1}{\sqrt{2}} |+z\rangle - \frac{1}{\sqrt{2}} |-z\rangle$, $|+y\rangle = \frac{1}{\sqrt{2}} |+z\rangle + \frac{1}{\sqrt{2}} |-z\rangle$ and $|-y\rangle = \frac{1}{\sqrt{2}} |+z\rangle - \frac{1}{\sqrt{2}} |-z\rangle$.

5. Rotation about z-Axis
Townsend problem 2.3 (page 61)  (2P)

6. Angular Momentum
Townsend problem 2.5 (page 61)
*Hint:* Use hint of problem 4.  (3P)

7. Rotation about y-Axis
Townsend problem 2.6 (page 61)  (3P)
8. Hermitian Operator
In quantum mechanics we represent physical quantities with an operator, in specific we use hermitian operators. The third postulate tells us that the only possible results of a measurement are eigenvalues of the corresponding operator. Since we measure in experiments real numbers (instead of complex numbers), we would expect the eigenvalues of the corresponding operator to be real. In this problem you show that this is the case if you describe physical quantities with hermitian operators.

Show that the eigenvalues of a hermitian operator are real. (2P)

9. Stern-Gerlach Experiments
Make sketches of the Stern-Gerlach experiments two to four and label in bra-ket notation the state of your Ag-atoms and at the outcome the probabilities. (2P)
Here is one example for the first SG experiment:

\[
\begin{align*}
\text{SG}_z &\quad |+z> \quad \text{prob. } |<+z|+z>|^2 \\
\text{SG}_z &\quad |-z> \quad \text{prob. } |<-z|+z>|^2
\end{align*}
\]

measured values for \(S_z\) are:
\[
\hat{J}_z |+z> = + \frac{\hbar}{2} \\
\hat{J}_z |-z> = - \frac{\hbar}{2}
\]