Homework Assignment #3

due: Wednesday, September 27, 1pm

1. Phaseshift (Townsend problem 1.9)
Given a state

\[ |\psi\rangle = \sum_{i=1}^{N} c_i |a_i\rangle \]

where \( |a_i\rangle \) are eigenvectors of \( \hat{A} \) with eigenvalues \( a_i \), i.e. \( \hat{A}|a_i\rangle = a_i |a_i\rangle \).
Show that neither the probability of obtaining the result \( a_i \) nor the expectation value \( \langle \hat{A} \rangle \) are affected by \( |\psi\rangle \rightarrow e^{i\phi}|\psi\rangle \), that is, by an overall phase change for the state \( |\psi\rangle \). (2P)

2. Taylor Series
Townsend problem 2.1
We used this identity in Eq.(2.32), i.e. the relation of a finite rotation operator and the corresponding angular momentum operator. (3P)

3. Angular Momentum - Hermitian Operator
Use Eq.(2.32) to show that we can conclude from \( \hat{R}(\phi) \) being a unitary operator that \( \hat{J}_z \) is a hermitian operator. (3P)

4. Commutation
Townsend problem 3.1 (4P)

5. Hermitian Operator
Townsend problem 3.8 (3P)

6. Angular Momentum - Commutation Relation (Townsend problem 3.10)
(a) What are the matrix representations of \( \hat{J}_x, \hat{J}_y \) and \( \hat{J}_z \) in the \( |+z\rangle, |-z\rangle \) basis?
(b) Verify explicitly through matrix multiplication that \( [\hat{J}_x, \hat{J}_y] = i\hbar \hat{J}_z \). (5P)