Homework Assignment #4

due: Wednesday, October 4, 1pm

1. Pauli Spin Matrices
   (a) Townsend problem 3.3
   (b) Townsend problem 3.4a
   (c) Show that $\sigma_x\sigma_y = i\sigma_z$, $\sigma_x\sigma_z = -i\sigma_y$ and $\sigma_y\sigma_z = i\sigma_x$.
      Hint: Use your results of (a) and (b).
   (d) Townsend problem 3.4b
      Hint: Use your results of (a) and (c). (4P)

2. Schwarz Inequality
   To derive the general uncertainty relation (Eq. (3.74)) the Schwarz inequality (Eq. (3.65)) was used. In this problem you will prove the Schwarz inequality.

   Townsend problem 3.7 but ignore his suggestion and use instead that $\langle \gamma | \gamma \rangle \geq 0$ for the specific vector $|\gamma\rangle = |\beta\rangle - \frac{\langle \alpha | \beta \rangle}{\langle \alpha | \alpha \rangle} |\alpha\rangle$. (2P)

3. Uncertainty Relation
   Townsend problem 3.9 (3P)

4. Eigenvalues and Eigenvectors
   Find the eigenvalues and corresponding eigenvectors of

   \[
   \begin{pmatrix}
   2 & 0.4 \\
   0.4 & 2
   \end{pmatrix}
   \] (2P)

5. Spin-1 Matrices
   Townsend problem 3.14
   Hint: Use the same approach as in §3.6 where the matrices of $\hat{S}_x$ and $\hat{S}_y$ were derived for spin-$\frac{1}{2}$ particles. (3P)

6. Spin-1 Eigenstates
   Townsend problem 3.15
   Hint: Use your result of problem 5 to represent $\hat{S}_x$. (3P)

7. Eigenstates of $\hat{S} \cdot \mathbf{n}$
   You will derive in this problem $| + \mathbf{n} \rangle$ and $| - \mathbf{n} \rangle$ for the spin-$\frac{1}{2}$ particle, which you used in the homework assignment #1 for the problems 1.3 - 1.7 of Townsend.

   Townsend problem 3.2
   Hint: Set up the matrix $\hat{S} \cdot \mathbf{n}$ using the equations (3.77), (3.88) and 3.89 and then determine its eigenvalues and corresponding eigenvectors. (3P)

1 which is true for any vector $|\gamma\rangle$