PHYS 334 Electromagnetic Theory II

In Class Exercise 12 — February 16, 2024 Name: Solutions

1. Using $\mathbf{k}\cdot\mathbf{r} = k_x\,\hat{\mathbf{x}} + k_y\,\hat{\mathbf{y}} + k_z\,\hat{\mathbf{z}}$ and $\nabla = \frac{\partial}{\partial x}\,\hat{\mathbf{x}} + \frac{\partial}{\partial y}\,\hat{\mathbf{y}} + \frac{\partial}{\partial z}\,\hat{\mathbf{z}}$, show that $\nabla e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} = i\mathbf{k}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$.

$$\vec{\nabla} e^{i(\vec{k}\cdot\vec{r}-\omega t)} = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right) e^{i(k_x x + k_y y + k_z z - \omega t)}$$

$$= \left(ik_x\hat{x} + ik_y\hat{y} + ik_z\hat{z}\right) e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$= i\vec{k}\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

2. Starting from the EM field energy density $u = \epsilon_0 E_0^2 \cos^2(\mathbf{k}\cdot\mathbf{r} - \omega t + \delta)$, show that $\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B} = cu\,\hat{\mathbf{k}}$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{1}{\mu_0}\left(E_0\cos(\vec{k}\cdot\vec{r}-\omega t+\delta)\,\hat{n}\right)\times\left(\frac{E_0}{c}\cos(\vec{k}\cdot\vec{r}-\omega t+\delta)(\hat{k}\times\hat{n})\right)$$

$$= \frac{1}{\mu_0 c}E_0^2\cos^2(\vec{k}\cdot\vec{r}-\omega t+\delta)\,\hat{k} \qquad [\hat{n}\times(\hat{k}\times\hat{n}) = \hat{k}]$$

$$= \frac{1}{\mu_0\varepsilon_0 c}\,\varepsilon_0 E_0^2\cos^2(\;)\,\hat{k}$$

$$= \frac{1}{\mu_0\varepsilon_0 c}u\hat{k} = cu\hat{k} \qquad \text{using } \frac{1}{\mu_0\varepsilon_0} = c^2$$