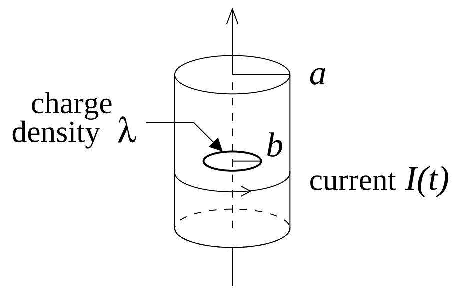
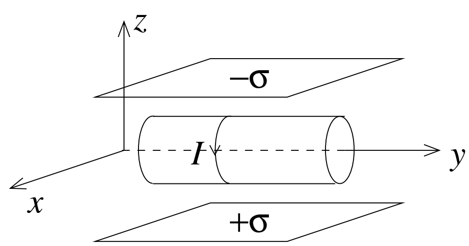
Worksheet for Final Exam

1. A solenoid of n turns per unit length and radius a carries a current that starts at zero and increases at a uniform rate: $I(t) = \dot{I}t$, with $\dot{I}$ constant. Within the solenoid is a ring of uniform charge density λ and radius b, as shown. The ring lies in a plane perpendicular to the solenoid axis, and is concentric with the solenoid.



Calculate the net torque on the ring.

2. A solenoid of n turns per unit length, radius a, and current I is centered on the y axis. Parallel plates with charge density $\pm\sigma$ sandwich the solenoid, as shown below.



(a) Calculate the electromagnetic energy density inside the solenoid.

(b) Calculate the electromagnetic momentum density inside the solenoid.

(c) In view of your answer to (b), what electromagnetic forces, if any, would act on the solenoid while the parallel plates are being discharged ($\pm\sigma \to 0$)?

3. A current $I(t) = \alpha t^2$ runs along the x axis in the $+x$ direction along a charge-neutral wire. Write down expressions for $V(\mathbf{r}, t)$ and $\mathbf{A}(\mathbf{r}, t)$ at the point $\mathbf{r} = z\,\hat{\mathbf{z}}$. You may leave your answer in terms of an integral, but every variable in the problem must either be a given (above), a physical constant, an integration variable, or something related to these by an auxiliary equation that you provide.

4.

(a) Expressing the fields in terms of the potentials

$$\mathbf{E} = -\vec{\nabla} V - \frac{\partial \mathbf{A}}{\partial t} \qquad \mathbf{B} = \vec{\nabla} \times \mathbf{A}$$

implies some of Maxwell's equations. Show which of Maxwell's equations follow from the above.

(b) Maxwell's equations (counting vector components) represent eight equations. How many are implied by the potential formulation?

(c) In the field formulation there are eight linear equations to determine six field components, which means more is implied by the equations than just the values of the **E** and **B** fields. Find an example of something implied by Maxwell's equations not related to the fields.

5. "Derive" the Lorentz force.

A particle with charge q is at rest in the frame $\bar{S}$, and so feels a force $\bar{\mathbf{F}} = q\bar{\mathbf{E}}$. $\bar{S}$ moves with velocity $\mathbf{v} = v\,\hat{\mathbf{x}}$ with respect to frame S. Transform the force and the fields to the frame S to get the force $\mathbf{F}$ in terms of the fields $\mathbf{E}$ and $\mathbf{B}$. Show this is equivalent to the Lorentz force.

6. Derive the wave equation for **B** in the absence of charge and current.

7. For a plane wave $\tilde{\mathbf{E}} = \tilde{E}_0 e^{i(\kappa \cdot \mathbf{r} - \omega t)} \hat{n}$

(a) Use Gauss's law for $\rho = 0$ to show that the $\tilde{\mathbf{E}}$ is transverse to the direction of propagation. (Hint: $e^{i\kappa \cdot \mathbf{r}} = e^{i\kappa_x x} e^{i\kappa_y y} e^{i\kappa_z z}$.)

(b) Show that $\tilde{\mathbf{E}}$ is a solution to the wave equation

$$\nabla^2 \tilde{\mathbf{E}} = \frac{1}{c^2} \frac{\partial^2 \tilde{\mathbf{E}}}{\partial t^2}$$

and derive the relation between κ, ω, and c.