Problem C

Here are a couple of situations where we wouldn't usually make a state space plot. But they are good for practice.

(a) Consider an object starting from rest and accelerating downward due to gravity alone (no air resistance). Make a plot (by hand is fine) of $\dot{y}$ versus y.

(b) Now consider the same situation except with air resistance. Make a plot that shows the trajectory from $t = 0$ into the time region where the object has reached terminal velocity.

Problem D

We're going to make both a state space plot for the attractor plus a Poincare section for a variety of γ. Consider the driven damped pendulum with $\omega = 2\pi$, $\omega_0 = 3\pi$, and $\beta = 3\pi/4$. Use initial condition $\phi(0) = -\pi/2$ and $\dot{\phi}(0) = 0$. Here are the steps.

1. Define `eqn` as before. But in the `NDSolve` command, there is an important difference: type

   ```
   soln = NDSolve[{eqn, φ[0]==-π/2, φ'[0]==0}, φ, {t, 0, 100}, MaxSteps → Infinity].
   ```

   The difference is in the middle: instead of defining the thing to solve for as `φ[t]`, define it as ϕ. I don't know why, but it makes a difference.

2. For the parametric plot, use the command

   ```
   ParametricPlot[{φ[t], φ'[t]} /. soln,{t, 30, 100}, AspectRatio→0.6]
   ```

   Notice how the `/. soln` part comes after the curly braces.

3. For the Poincare plot, use the command

   ```
   list = Table[{φ[t], φ'[t]} /. soln, {t, 30, 99, 1}];
   ListPlot[list]
   ```

   Similar syntax for the `/. soln`. This takes values in increments of 1 from 30 to 99. For the Poincare plot, if you are seeing fewer points than you expect, it can be because choosing integer t values isn't optimal (it could be the place where the different phase space curves are close together). So there's no reason not to try going from 30.4 to 99.4 in steps of 1, for example, in case it gives more "typical" data.

So with that procedure, make the state space plot and the Poincare section for the following, and comment whether the results are what you expected.

(a) $\gamma = 1.06$; (b) $\gamma = 1.078$; (c) $\gamma = 1.081$; (d) $\gamma = 1.085$;

(e) Any other γ value that seems to give you a nice chaotic behavior. Play around!