Bucknell UNIVERSITY

# Theory of Computation
CSCI 341, Fall 2016

# Homework 1
# Due 2016-09-05

**(2pt) Problem 1**

What are these sets? Write them using braces, commas, and numerals only.

1. $(\{1,3,5\} \cup \{3,1\}) \cap \{3,5,7\}$
2. $\bigcup\{\ \{3\}, \{3,5\}, \bigcap\{\{5,7\}, \{7,9\}\}\ \}$
3. $(\{1,2,5\} - \{5,7,9\}) \cup (\{5,7,9\} - \{1,2,5\})$
4. $2^{\{7,8,9\}} - 2^{\{7,9\}}$
5. $2^{\emptyset}$

**(2pt) Problem 2**

What are these sets? Write them using braces, parentheses, commas, and numerals only.

1. $\{1\} \times \{1,2\} \times \{1,2,3\}$
2. $\emptyset \times \{1,2\}$
3. $2^{\{1,2\}} \times \{1,2\}$

**(2pt) Problem 3**

Let $R = \{(a,b), (a,c), (c,d), (a,a), (b,a)\}$. What is $R \circ R$, the composition of $R$ with itself? What is $R^{-1}$, the inverse of $R$? Is $R$, $R \circ R$, or $R^{-1}$ a function?

**(2pt) Problem 4**

Let $f : A \to B$. Let $R_f$ be the binary relation on $A$ defined as

$$xR_f y \text{ if and only if } f(x) = f(y).$$

Prove that $R_f$ is an equivalence relation.

**(2pt) Problem 5**

Let $A$ be a non-empty finite set and let $f : A \to A$. We have seen the definition of a *cycle* in a relation $R$. The function $f$ can be seen as its corresponding relation $R_f$. Prove that $R_f$ contains a cycle. *(Hint: your proof can be the description of an algorithm building such a cycle, together with a clear explanation of the algorithm's correctness.)*