Bucknell UNIVERSITY

Theory of Computation
CSCI 341, Fall 2016

Homework 2
Due 2016-09-23

(2pt) Problem 1

Following the spirit of the algorithm converting regular expressions to NFAs, propose an extension of this algorithm to convert regular expressions of the form $E = E_1^+$ to NFAs. Draw the conversion **and** define it formally, by assuming E_1 is recognized by an NFA $(Q_1, \Sigma, \Delta_1, s_1, F_1)$, define an NFA $N = (Q, \Sigma, \Delta, s, F)$ recognizing E_1^+.

SOLUTION Let $M_1 = (Q_1, \Sigma, \Delta_1, s_1, F_1)$ be an NFA recognizing the regexp E_1. We define the following NFA M recognizing E_1^+ by defining it as

$$M = (Q, \Sigma, \Delta, s, F)$$

with

- $Q = Q_1$
- $s = s1$
- $\Delta = \Delta_1 \cup \{(f, \varepsilon, s_1) \mid f \in F_1\}$
- $F = F_1$

(3pt) Problem 2

Convert the following regular expressions to NFAs using the procedure we saw in class, and then convert the NFAs to DFAs using the other procedure we saw in class.

1. $a^*(a(b^*))b$
2. $(a \cup b)^* b(aa)b$
3. $a(b^+)(a \cup b)^*(b^+)a$

(2.5pt) Problem 3

Exercise 4.7 from Sipser, page 211. Maximum credit for a clear and complete proof.

SOLUTION Let B be the set containing all the infinite sequences of booleans. For example, elements of B could be sequence s like:

$$s = 0000000000\cdots$$

$$s = 1111111111\cdots$$

$$s = 01010101\cdots$$

$$s = 0110001110\cdots$$

Let us call call $s[i]$ the i-th digit of s.

We want to prove that B is *uncountable*. Let's prove it by contradiction.

Let us assume that B is countable. It means that there exists an isomorphism $f : \mathbb{N} \rightarrow B$. Let us define a particular sequence d (the diagonal sequence) using the function f.

$$d[i] = \overline{f(i)[i]}$$

The function f is an bijection. Therefore, that there exists an integer k such that $f(k) = d$. The contradiction appears when we inspect the value of $d[k]$. There are two possibilities: $d[k] = 0$ or $d[k] = 1$.

- Suppose $d[k] = 0 \Rightarrow \overline{f(k)[k]} = 0 \Rightarrow f(k)[k] = 1 \Rightarrow d[k] = 1$. It's absurd!
- Suppose $d[k] = 1 \Rightarrow \overline{f(k)[k]} = 1 \Rightarrow f(k)[k] = 0 \Rightarrow d[k] = 0$. It's absurd!

This is a contradiction! In conclusion, it cannot be true that B is countable.

(2.5pt) Problem 4

Problem 1.32 from Sipser, page 88.