

**Homework Assignment #3 – due via Moodle at 11:59 pm on Thursday, Oct. 2, 2025**

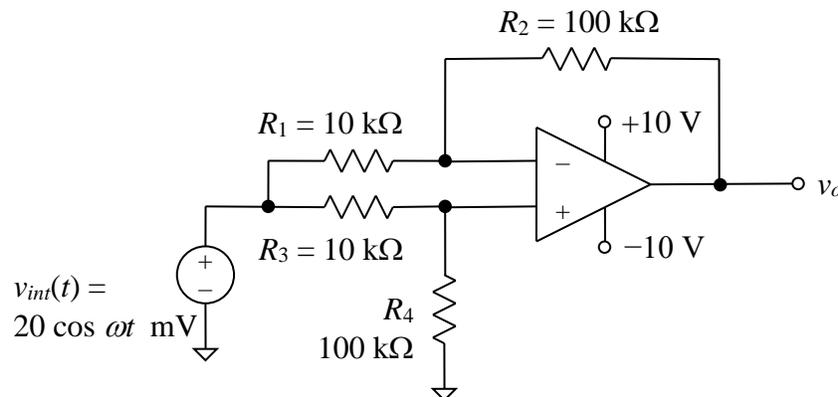
***Instructions, notes, and hints:***

You may make reasonable assumptions and approximations to compensate for missing information, if any. Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

Note that the first few problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

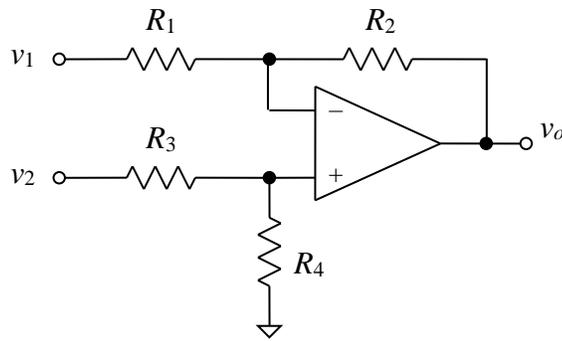
***Graded Problems:***

- The circuit shown below models a diff amp that has its input terminals shorted together so that the differential-mode input voltage is zero. However, the diff amp does experience a common-mode interference signal  $v_{int}(t)$  at its inputs. The interference frequency is  $f = 100$  kHz. Nominal resistor values are shown, but the resistor tolerance is 5%. Assuming that the actual resistor values are no more than  $\pm 5\%$  away from their respective nominal values, find the worst-case (largest possible) magnitude of the common-mode output voltage  $v_o$ .



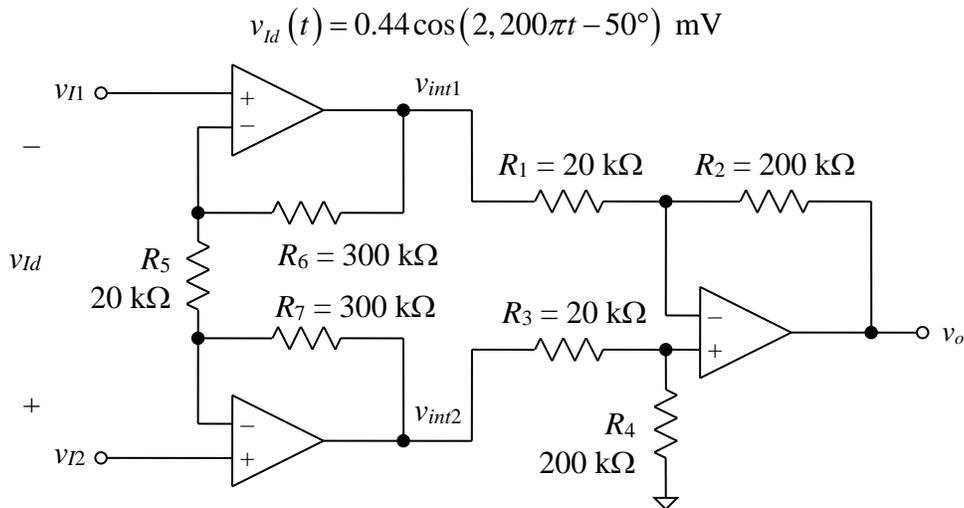
- Suppose that you have been asked to design a new digital data link, and the system needs a basic diff amp like the one shown at the top of the next page. The equivalent resistance of the load connected to the output of the amplifier is 50 k $\Omega$ . The differential-mode gain must be around 20 V/V, and the differential-mode input resistance must be  $R_{id} = 50$  k $\Omega$ . The power supply voltages are to be  $\pm 10$  V. The selected op-amp has a maximum rated output current of 30 mA, a maximum input bias current of 20 nA, and a peak input offset voltage magnitude of 1.5 mV. Find the maximum allowable tolerance for the resistors  $R_1$  through  $R_4$  to ensure that the CMRR of the diff amp would be at least 60 dB (which is not very impressive).

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**Circuit diagram for Graded Prob. 2**

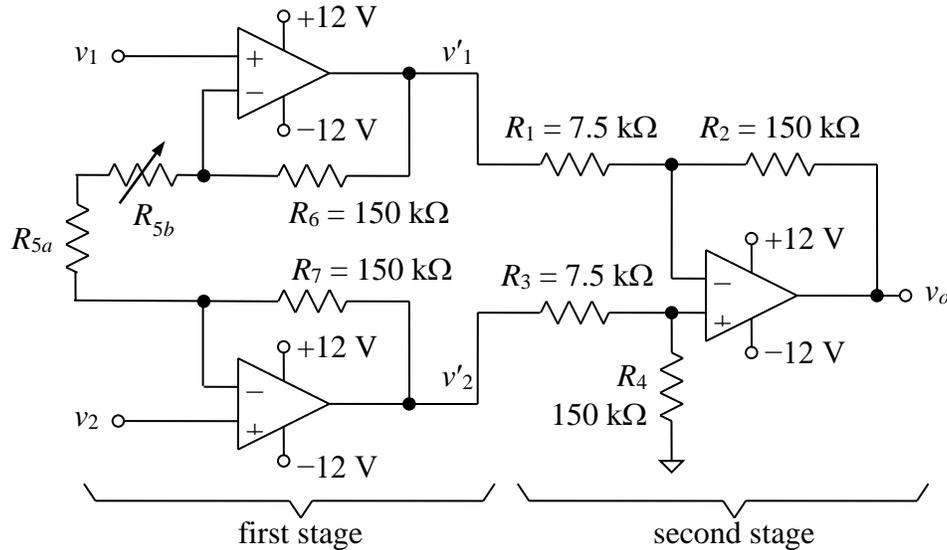
3. The resistors in the instrumentation amplifier shown below are labeled with their nominal values. A sinusoidal differential-mode voltage with the waveform given by the expression below is applied across the input terminals of the amplifier (between nodes  $v_{I1}$  and  $v_{I2}$ ). The common-mode voltage gain of the second stage is 0.028 V/V. Find the resulting peak-to-peak differential intermediate voltage  $v_{int2} - v_{int1}$  and the peak-to-peak differential output voltage  $v_{od}$ . The power supply voltages are  $\pm 15$  V but are not shown in the diagram to improve clarity.



4. Referring to the instrumentation amplifier considered in the previous problem, suppose that nearby building wiring causes a common-mode periodic signal with a peak-to-peak voltage of 120 mVpp and a frequency of 60 Hz to appear at the two inputs  $v_{I1}$  and  $v_{I2}$ . All resistor values are the same as in the previous problem. The common-mode gain of the second stage is also the same. Ignoring the differential-mode input voltage, find the peak-to-peak common-mode node voltages at the  $v_{int1}$ ,  $v_{int2}$ , and  $v_o$  nodes.

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5. The instrumentation amplifier depicted below uses 2% tolerance resistors in the first stage and 0.1% tolerance resistors in the second stage. The circuit is unusual in that resistor  $R_5$  has been split into a fixed portion  $R_{5a}$  and a variable portion  $R_{5b}$ . The variable resistor can have a minimum value of zero and a maximum value  $R_{5bmax}$ . Find the required values for  $R_{5a}$  and  $R_{5bmax}$  so that the overall nominal differential-mode gain varies from 40 to 100 V/V with roughly 5% accuracy or better. Also find the lowest possible overall CMRR value (expressed in dB) obtained as  $R_{5b}$  is varied over its full range of values. That is, find the worst-case CMRR obtained over the full range of differential-mode gains from  $A_d = 40$  to 100 V/V.

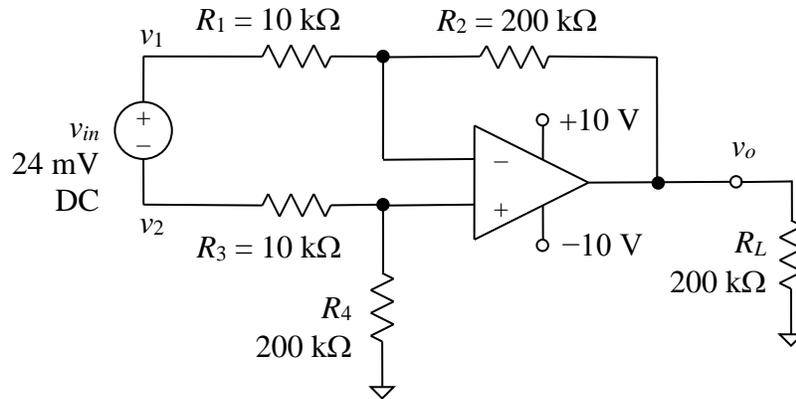


**Ungraded Problems:**

The following problems will not be graded. They are intended to serve as practice problems and examples. The solutions will be posted along with the solutions to the graded problems. You should attempt to solve them on your own and then check the solutions afterwards. Do not give up too quickly if you struggle with any of them. Move on to a different problem and then come back to the difficult one after a few hours.

1. The resistors in the diff amp depicted on the next page are labeled with their nominal (ideal) values. Each one has a 2% tolerance. The actual resistor values (to four-digit accuracy) are  $R_1 = 10.10 \text{ k}\Omega$ ,  $R_2 = 196.4 \text{ k}\Omega$ ,  $R_3 = 10.14 \text{ k}\Omega$ , and  $R_4 = 198.8 \text{ k}\Omega$ . A sensor is generating a quasi-DC (i.e., very slowly varying) input voltage that currently has a value of 24 mV. The sensor is connected to the amplifier via a long cable in a noisy environment. The building's AC wiring causes a common-mode input voltage expressed as  $v_{Icm}(t) = 0.140 \cos(377t) \text{ V}$  to appear at each input of the amplifier. Note that  $|v_{in}| < |v_{Icm}|_{pk}$ . Find:
- the nominal differential-mode gain (i.e., the gain assuming ideal resistor values).
  - the actual differential-mode gain  $A_d$  (using actual resistor values).
  - the actual common-mode gain  $A_{cm}$ .
  - the actual CMRR (in dB).
  - the differential-mode output voltage  $v_{od}$ .
  - the common-mode output voltage  $v_{ocm}$ .

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**Circuit diagram for Ungraded Prob. 1**

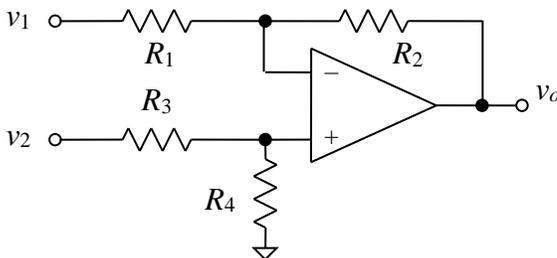
- Suppose that the resistors in the diff amp in the previous problem are replaced with new ones that have 5% tolerance. The actual resistor values are now  $R_1 = 10.42 \text{ k}\Omega$ ,  $R_2 = 191.0 \text{ k}\Omega$ ,  $R_3 = 10.38 \text{ k}\Omega$ , and  $R_4 = 205.6 \text{ k}\Omega$ . Repeat parts **a** through **f** of the previous problem. Comment on the relative changes in the differential-mode gain and the common-mode gain; that is, how do the magnitudes of the changes compare to each other, and why?
- In the supplemental reading “Real-World Performance of Difference Amplifiers,” it is shown that the worst-case CMRR for a diff amp occurs when

$$R_1 = (1 - \varepsilon)R_{1nom} \quad R_2 = (1 + \varepsilon)R_{2nom} \quad R_3 = (1 + \varepsilon)R_{3nom} \quad R_4 = (1 - \varepsilon)R_{4nom},$$

where the “nom” subscript indicates the nominal (ideal) value of the resistor and where  $\varepsilon$  is the fractional tolerance (e.g.,  $\varepsilon = 0.05$  corresponds to 5%). The same worst-case CMRR is found if the “+” signs are replaced with “-” signs and vice versa above. Using the worst-case resistor values given above, show that the approximation given below left is valid regardless of the nominal resistor values. In a diff amp, the nominal resistor values have the relationship given below right.

$$1 - \frac{R_2}{R_1} \frac{R_3}{R_4} \approx 4\varepsilon \qquad \frac{R_{2nom}}{R_{1nom}} \frac{R_{3nom}}{R_{4nom}} = 1$$

- For the basic diff amp circuit depicted below, use a derivation like the one outlined in the supplemental reading “Real-World Performance of Difference Amplifiers” to show that a good approximation of the worst-case common-mode gain is given by the expression for  $A_{cm}$  given below. You may assume that the resistor value ratios satisfy the given approximation.

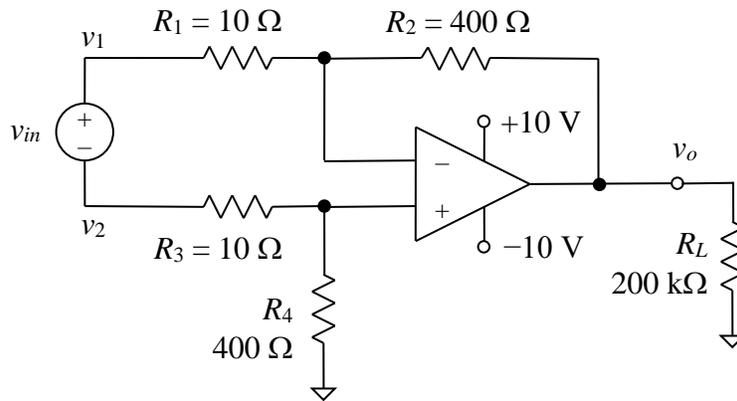


$$A_{cm} \approx 4\varepsilon \frac{R_2}{R_1 + R_2}$$

$$\frac{R_4}{R_3} \approx \frac{R_2}{R_1}$$

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5. The op-amp in the diff amp circuit depicted below has an output current magnitude limit of 10 mA. The resistor values used in the circuit are ridiculously small; this problem helps to illustrate why. Input voltage  $v_{in}$  can be as high as  $\pm 250$  mV before the output saturates at one of the power supply limits ( $\pm 10$  V here). However, output current limiting prevents the output voltage from reaching that level. If  $v_{in} = -250$  mV (DC), find the resulting voltage  $v$  across the input terminals of the op-amp (positive side of  $v$  at the “+” terminal). *Hint:* Check whether the load connected to the output of the circuit has a high enough equivalent resistance so that the current flowing through it can be considered negligible.



6. The instrumentation amplifier shown below was accidentally assembled with the wrong value for resistor  $R_7$ ; its value should have been  $100 \text{ k}\Omega$ . All resistors have 5% tolerance; the values shown are the nominal ones. If the common-mode gain  $A_{cm2}$  of the second stage is  $0.0044 \text{ V/V}$ , find the differential-mode and common-mode gains of the full amplifier to reasonably good approximations (accurate to  $\pm 5\%$  or so). Also find the CMRR expressed in decibels.

