Homework Assignment \#2 - due via Moodle at 11:59 pm on Friday, Feb. 2, 2024

## Instructions, notes, and hints:

You may make reasonable assumptions and approximations in order to compensate for missing information, if any. Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

Unless otherwise specified, you may assume that all BJTs are at room temperature, the emission coefficient $n=1, V_{B E}=0.7 \mathrm{~V}$ (quiescent value), and $\left.V_{C E}\right|_{\text {sat }}=0.2 \mathrm{~V}$. If the Early voltage $V_{A}$ is not specified, you may ignore its effects. For now, unless otherwise specified, capacitors can be assumed to have values large enough that they act as shorts at the operating frequency.

The first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

## Graded Problems:

1. The emitter follower shown below has a bipolar power supply with $\pm 5 \mathrm{~V}$. After finding the quiescent collector current, find the items listed below. You may assume that $\beta \approx 150$.
a. Symbolic (algebraic) expression in terms of the resistor values and the BJT parameters $\beta, r_{\pi}$, and/or $g_{m}$ for the small-signal voltage gain $A_{v}=v_{o} / v_{i n}$. Also find its numerical value.
b. Symbolic expression for and numerical value of the input resistance $R_{\text {in }}$ seen by the signal source represented by $v_{\text {sig }}$ and $R_{\text {sig }}$.
c. Symbolic expression for and numerical value of the output resistance $R_{\text {out }}$ seen by the load represented by $R_{L}$.

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2. For the common-base amplifier shown below, find symbolic (algebraic) expressions in terms of the resistor values and the BJT parameters $\beta, r_{\pi}$, and/or $g_{m}$ for the following:
a. Small-signal voltage gain $A_{v}=v_{o} / v_{\text {in }}$.
b. Input resistance $R_{\text {in }}$ seen by the signal source represented by $v_{\text {sig }}$ and $R_{\text {sig }}$.
c. Output resistance $R_{\text {out }}$ seen by the load represented by $R_{L}$.

You may assume that the BJT is biased into the active region and that the Early effect can be ignored (i.e., $V_{A} \rightarrow \infty$ or, equivalently, $r_{o} \rightarrow \infty$ ).

3. Find a symbolic expression for the small-signal constraint on $v_{\text {in }}$ for the common base amplifier considered in the previous problem. That is, find the maximum value that $v_{i n}$ can have to ensure that the small-signal condition $\left|v_{b e}\right| \ll n V_{T}$ is satisfied.
4. The common emitter amplifier depicted below has a bipolar power supply. The quiescent collector current is $I_{C}=2.1 \mathrm{~mA}$. Use a small-signal model to find numerical values for the quantities listed below. Find symbolic (algebraic) expressions first and then substitute numerical values. You may assume that $\beta \approx 150$.
a. Small-signal voltage gain $A_{v}=v_{o} / v_{i n}$.
b. Input resistance $R_{\text {in }}$ seen by the signal source represented by $v_{\text {sig }}$ and $R_{\text {sig }}$.
c. Output resistance $R_{\text {out }}$ seen by the load represented by $R_{L}$.

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5. Find the numerical value of the small-signal output resistance of the source follower depicted below as seen by the load $R_{L}$. The MOSFET has the parameters shown next to the diagram. Note that you will first need to find the quiescent drain current $I_{D}$. Ignore the channel-length modulation effect. The capacitors have negligible reactances at the signal frequency.


## Ungraded Problems:

The following problems will not be graded. However, you should attempt to solve them on your own and then check the solutions. Try not to give up too quickly if you struggle to solve any of them. Move on to a different problem and then come back to the difficult one after a few hours.

1. The common emitter amplifier shown below uses the collector-to-base biasing method $\left(R_{1}\right)$. The quiescent collector current is close to $500 \mu \mathrm{~A}$, and the quiescent collector voltage is close to 6.0 V . (Try to verify those values yourself!) Find the items listed below. You may assume that $\beta \approx 150$.
a. Symbolic (algebraic) expression in terms of the resistor values and the parameters $\beta$, $r_{\pi}$, and/or $g_{m}$ for the small-signal voltage gain $A_{\nu}=v_{o} / v_{i n}$ and its numerical value.
b. Symbolic expression for and numerical value of the input resistance $R_{\text {in }}$ seen by the signal source represented by $v_{\text {sig }}$ and $R_{\text {sig }}$.
c. Symbolic expression for and numerical value of the output resistance $R_{o}$ seen by the load represented by $R_{L}$.

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2. Find a symbolic expression for the small-signal constraint on $v_{i n}$ expressed as a function of the various component values and BJT parameters for the emitter follower below. That is, find the maximum value that $v_{i n}$ can have to ensure that the small-signal condition $\left|v_{b e}\right| \ll$ $n V_{T}$ is satisfied. Recall that the small-signal condition allows the approximation below right to apply and is therefore the basis for the hybrid-pi model of the BJT.


$$
e^{v_{b e} / n V_{T}} \approx 1+\frac{v_{b e}}{n V_{T}}
$$

3. The circuit below illustrates one way to drive a low-impedance load efficiently using a common-emitter amplifier. (The output resistance of a traditional CE amp equals the value of the collector resistor $R_{C}$.) The transformer may be assumed to be ideal. The parallel lines between the inductors indicates that they are wound on an iron core. The transformer's primary winding replaces the usual collector resistor $R_{C}$. The bias design has already been completed for the circuit. Explain why the quiescent collector voltage $V_{C}$ (the node voltage at the collector) is equal to 6 V , and show that the quiescent collector current $I_{C}$ is around 10.5 mA . The exact value of $\beta$ is unknown but lies in the range $50-150$.

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4. Assuming that the transformer in the amplifier circuit of the previous problem is ideal, show that the small-signal equivalent resistance looking into the primary terminals ( $a$ and $b$ ) from the collector is equal to $1,250 \Omega$. That is, show that the transformer converts the load impedance on the secondary side to a higher value on the primary side equal to $N^{2} R_{L}$, where $N$ is the primary-to-secondary turns ratio. Hint: The ratio of the primary voltage to the secondary voltage is $5: 1$, but the ratio of the primary current to the secondary current is $1: 5$.
5. Use the results of the previous two problems to estimate the small-signal voltage gain $v_{o} / v_{\text {in }}$ of the amplifier, assuming that the transformer is ideal. The exact value of $\beta$ is unknown but lies in the range 50-150. Hint: Find the intermediate gain $v_{a b} / v_{i n}$ and then use that result to find the desired gain $v_{o} / v_{i n}$.
6. In the common-emitter amplifier shown below, assume that the bias network is designed so that the quiescent values of $V_{C}$ and $V_{E}$ are set to $0.5 V_{C C}$ and $0.25 V_{C C}$, respectively. The signal source is represented by the circuit elements labeled $v_{s i g}$ and $R_{\text {sig }}$. Show that the small-signal open-circuit voltage gain magnitude (for $R_{L} \rightarrow \infty$ ) under these conditions is given by

$$
\left|A_{v o}\right|=\left|\frac{v_{o}}{v_{i n}}\right|=\frac{V_{C C}}{2 n V_{T}} .
$$

Also calculate the numerical values of the open-circuit gain magnitude at room temperature for $V_{C C}=12 \mathrm{~V}, 5 \mathrm{~V}$, and 1.8 V , and briefly discuss the implications. Hint: Consider what the quantities $I_{C} R_{C}$ and $I_{E} R_{E}$ are equal to.

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7. (Adapted from Prob. 7.136 in Sedra \& Smith, $8^{\text {th }}$ ed.): The circuit shown below is called a bootstrapped follower. It uses a clever modification of the traditional emitter follower to greatly increase the input resistance without significantly affecting the voltage gain. For follower shown below:
a. Find the quiescent collector current and the values of $g_{m}, r_{e}$, and $r_{\pi}$, assuming that $\beta=$ 100.
b. Replace the BJT with the T small-signal model, but omit $r_{o}$. Find a symbolic expression for the input resistance $R_{i n}$, and then find the numerical value.
c. Find a symbolic expression for the overall voltage gain $v_{o} / v_{s i g}$, and then find the numerical value.
d. Repeat parts (b) and (c) for the case when capacitor $C_{B}$ is open-circuited. Compare the results with those obtained in (b) and (c) and discuss the implications.

Hints for parts (b) and (c): The T model for BJTs is discussed in Section 7.2.2 of the textbook. It is an alternative to the hybrid-pi model and is sometimes more useful for certain circuit topologies. Also, you will probably have to evaluate the quantity $\left(1 / r_{e}\right)-g_{m}$. If so, be careful because $1 / r_{e}$ and $g_{m}$ are approximately (but not exactly) equal. A good approach is to apply the result below right.


$$
\frac{1}{r_{e}}-g_{m}=\frac{\beta+1}{r_{\pi}}-\frac{\beta}{r_{\pi}}=\frac{1}{r_{\pi}}
$$

