Homework Assignment \#6 - due via Moodle at 11:59 pm on Friday, Mar. 8, 2024

## Instructions, notes, and hints:

You may make reasonable assumptions and approximations to compensate for missing information, if any. Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

Unless otherwise specified, you may assume that all BJTs are at room temperature, the emission coefficient $\eta=1, V_{B E}=0.7 \mathrm{~V}$ (quiescent value), and $\left.V_{C E}\right|_{\text {sat }}=0.2 \mathrm{~V}$. If the Early voltage $V_{A}$ is not specified, you may ignore its effects. For now, unless otherwise specified, capacitors can be assumed to have values large enough that they act as shorts at the operating frequency.

The first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

## Graded Problems:

1. Determine the minimum value of the potentiometer $R_{\text {pot }}$ in the circuit shown below required to allow cancellation of an input offset voltage Vos of up to $\pm 5 \mathrm{mV}$ caused by a mismatch in the junction areas of the two BJTs. The two collector resistors have been laser trimmed so that the difference in their values is insignificant. The average value of $\beta$ is 200 . If you need a value for $\alpha$ [equal to $\beta /(\beta+1)$ ], you may use the average value. The value of $R_{E E}$ is large enough that its effect on the input offset voltage (and the compensation of $V_{O S}$ ) can be ignored. The resistance between the wiper of the potentiometer and its left terminal is $x R_{p o t}$, and the resistance between the wiper and the right terminal is $(1-x) R_{p o t}$. You may ignore the Early effect in $Q_{1}$ and $Q_{2}$. Hint: It might be helpful to read the definitions of $I_{S}$ (the scale or saturation current) found on p. 162 and p. 311 of the textbook (Sedra \& Smith, $8^{\text {th }}$ ed., Sec. 6.1.2). The latter definition is the more useful one in the context of BJTs.

(continued on next page)
2. This problem examines a practical issue that must be considered when testing diff amps with current mirror loads. As shown below, a load resistance $R_{L}$ has been connected to the output node of a diff amp for test purposes. The value of $R_{E E}$ is large enough that its effect on the quiescent bias currents can be ignored. $Q_{1}$ and $Q_{2}$ have $V_{A}=50 \mathrm{~V}, \beta=200$, and $\left.V_{C E}\right|_{\text {sat }}=$ $0.2 \mathrm{~V} . Q_{3}$ and $Q_{4}$ have $V_{A}=50 \mathrm{~V}, \beta=100$, and $\left.V_{E C}\right|_{\text {sat }}=0.2 \mathrm{~V}$. Account for the Early effect for small-signal analysis, but ignore it for large-signal and bias analyses. You will need to understand the material covered in Sec. 9.5 of the textbook (Sedra \& Smith, $8^{\text {th }}$ ed.) and the related lecture material to solve this problem. Find:
a. An expression for the small-signal voltage gain with a finite load resistance $R_{L}$ present.
b. The maximum value that $R_{L}$ can have to ensure that $Q_{2}$ and $Q_{4}$ operate in the active region even if $\left|V_{I N}\right|>4 n V_{T}$.
c. Comment on how the results of parts a and b place conflicting pressures on the selection of a value for $R_{L}$.

3. [adapted from Prob. 9.97 of Sedra \& Smith, $8^{\text {th }}$ ed.] BJT diff amps with current mirror loads suffer from a systematic offset voltage effect. It occurs because the quiescent collector currents of $Q_{1}$ and $Q_{2}$ are both equal to $\alpha I_{E E} / 2$, but $I_{C 1}=I_{C 3}+I_{B 3}+I_{B 4}$, so $I_{C 4}<I_{C 2}$ (quiescent currents) by a small amount. The difference must flow through the load, which results in a non-zero output voltage. Referring to the circuit diagram in the previous problem, show that the input offset voltage related to this effect is given by the expression below, where $\beta_{P}$ is the beta value of $Q_{3}$ and $Q_{4}$. Assume that the circuit is otherwise balanced. Hints: 1) If $v_{I N}=0$ and the offset effect were not present, then $i_{C 2}=i_{C 4}$, and no current would flow through the parallel combination of $r_{o 2}, r_{o 4}$, and $R_{L}$, which would yield $v_{O}=0$. (In this context, collector currents $i_{C 2}$ and $i_{C 4}$ are the currents that are generated by the dependent current sources in the hybrid pi model and exclude the currents through $r_{o 2}$ and $r_{o 4}$.) 2) Because $i_{C 2}$ is not exactly equal to $i_{C 4}$, some current flows through the parallel combination of $r_{o 2}, r_{o 4}$, and $R_{L}$, even if $v_{I N}=0$, which creates a non-zero output voltage. 3) Note that $2 / \beta_{P} \ll 1$.

$$
V_{O S} \approx-\frac{2 \eta V_{T}}{\beta_{P}}
$$

## Ungraded Problems:

The following problems will not be graded. However, you should attempt to solve them on your own and then check the solutions. Try not to give up too quickly if you struggle to solve any of them. Move on to a different problem and then come back to the difficult one after a few hours.

1. The diagram below is the small-signal model of the output port of a common-emitter amplifier and its load. Show that the output voltage is given by the symbolic expression shown next to the diagram. Briefly explain the significance of the frequency-dependent part of the expression (the fraction to the right of $R_{C} \| R_{L}$ ).


$$
v_{o}=-g_{m} v_{b e}\left(R_{C} \| R_{L}\right) \frac{j \omega\left(R_{C}+R_{L}\right) C_{o}}{1+j \omega\left(R_{C}+R_{L}\right) C_{o}}
$$

2. For the partial amplifier small-signal model considered in the previous problem, find the cutoff frequency (in Hz) associated with capacitor $C_{o}$. Then, assuming that $v_{b e}=v_{i n}$, calculate the magnitude of the midband voltage gain $v_{o} / v_{i n}$ and the magnitude of the voltage gain at the cut-off frequency. Assume that the amplifier has a quiescent current of $I_{C}=2.0 \mathrm{~mA}$, the emission coefficient is $\eta=1$, and $V_{T}=25 \mathrm{mV}$.
