ECEG 390 Theory and Applications of Electromagnetics Spring 2026

Policies and Review Topics for Exam #3

The following policies will be in effect for the exam. They will be included in a list of instructions and policies on the first page of the exam:

1. You will be allowed to use a non-wireless enabled calculator, such as a TI-89.
2. You will be allowed to use up to **three** 8.5 × 11-inch two-sided handwritten help sheets. No photocopied material or copied and pasted text or images are allowed. If there is a table or image from the textbook or some other source that you feel would be helpful during the exam and that is not included on the table and formula sheet that I will provide, please notify me.
3. All help sheets will be collected at the end of the exam but will be returned to you either immediately or soon after the exam.
4. Use of a help sheet that is not completely handwritten will result in an automatic 5-point score reduction. Help sheets that are handwritten on a tablet and then printed are acceptable.
5. If you begin the exam after the start time, you must complete it in the remaining allotted time. However, you may not take the exam if you arrive after the first student has completed it and left the room. The latter case is equivalent to missing the exam.
6. **You may not leave the exam room without prior permission except in an emergency or for an urgent medical condition. Please use the restroom before the exam.**

The exam will begin at 3:00 pm on Thursday, April 2 in Breakiron 165. You will have until 4:50 pm to complete the exam.

The following is a list of topics that could appear in one form or another on the exam. Not all of these topics will be covered, and it is possible that an exam problem could cover a detail not specifically listed here. However, this list has been made as comprehensive as possible. **You should be familiar with the topics on the review sheets for the previous exams as well.**

Although significant effort has been made to ensure that there are no errors in this review sheet, some might nevertheless appear. The textbook is the final authority in all factual matters, unless errors have been specifically identified there. You are ultimately responsible for obtaining accurate information when preparing for the exam.

Antenna analysis (determination of radiation fields from current distributions)

- all time-varying currents act as sources and potentially can radiate EM waves. Examples:
  - antennas
  - the sun
  - lightning
  - sparks and other arc flashes
  - time-varying currents flowing in circuits (e.g., address bus in a computer, video signals, noise on power leads/bus)
  - charged particles accelerated by earth's magnetic field

- position vectors:
  - **R** (unprimed) defines observation point (distant point where E-field is calculated)
  - **R′** (primed) defines point on antenna
  - spherical coordinates: $\mathbf{R} = \hat{\mathbf{R}}R$, where direction of $\hat{\mathbf{R}}$ is a function of θ and ϕ
  - cylindrical coordinates: $\mathbf{R} = \hat{\mathbf{r}}r + \hat{\mathbf{z}}z$, where direction of $\hat{\mathbf{r}}$ is a function of ϕ
  - rectangular coordinates: $\mathbf{R} = \hat{\mathbf{x}}x + \hat{\mathbf{y}}y + \hat{\mathbf{z}}z$
- exact electric and magnetic fields radiated by Hertzian dipole (infinitesimally short filament of current with "length" l and uniform current distribution, located at origin, and aligned along z-axis; magnitude of input current $\widetilde{I}_{in}$ is in peak units):

$$\widetilde{\mathbf{E}} = \hat{\mathbf{R}}\frac{\widetilde{I}_{in}k^2 l}{4\pi}\eta e^{-jkR}\left[\frac{2}{(kR)^2} - \frac{j2}{(kR)^3}\right]\cos\theta + \hat{\boldsymbol{\theta}}\frac{\widetilde{I}_{in}k^2 l}{4\pi}\eta e^{-jkR}\left[\frac{j}{kR} + \frac{1}{(kR)^2} - \frac{j}{(kR)^3}\right]\sin\theta$$

$$\widetilde{\mathbf{H}} = \hat{\boldsymbol{\varphi}}\frac{\widetilde{I}_{in}k^2 l}{4\pi}e^{-jkR}\left[\frac{j}{kR} + \frac{1}{(kR)^2}\right]\sin\theta$$

- far field criterion ($kR >> 1$ for Hertzian dipoles)
- far fields of Hertzian dipole (uniform current distrib. in peak units) of length l:

$$\widetilde{\mathbf{E}} = \hat{\boldsymbol{\theta}}\frac{jk\eta\widetilde{I}_{in}l}{4\pi R}e^{-jkR}\sin\theta, \quad \widetilde{\mathbf{H}} = \hat{\boldsymbol{\varphi}}\frac{jk\widetilde{I}_{in}l}{4\pi R}e^{-jkR}\sin\theta$$

Common characteristics of *far*-field expressions (for any antenna orientation or location unless otherwise specified):

- the $\frac{e^{-jkR}}{R}$ factor, which implies spreading spherical waves (true for *all* antennas)
- propagation in $\hat{\mathbf{R}}$ direction (if antenna is centered at origin) (true for *all* antennas); the direction is directly away from the antenna
- speed of propagation is $\frac{1}{\sqrt{\mu\varepsilon}}$ (speed in surrounding medium; true for *all* transverse electromagnetic (TEM) waves)
- electric and magnetic fields are proportional to input current (true for *all* antennas fed by a transmission line)
- $\widetilde{\mathbf{E}} \perp \widetilde{\mathbf{H}}$, $\widetilde{\mathbf{E}} \perp \mathbf{S}_{av}$, and $\widetilde{\mathbf{H}} \perp \mathbf{S}_{av}$ (where $\mathbf{S}_{av}$ = time-average Poynting vector; true for *all* TEM waves; the "transverse" in TEM refers to orthogonality of **E** and **H** to **S**)
- electric and magnetic fields are in phase if η is purely real (true for *all* TEM waves)
- $\frac{|\widetilde{\mathbf{E}}|}{|\widetilde{\mathbf{H}}|} = \eta$ in a lossless medium (true for *all* TEM waves)
- For Hertzian dipoles and all straight-wire antennas aligned along the z-axis, the far electric field is θ-directed, and the far magnetic field is ϕ-directed.

Time-average Poynting vector

- definition: $\mathbf{S}_{av} = \frac{1}{2}\text{Re}\left\{\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^*\right\}$, if electric and magnetic fields are in peak units and expressed as phasors; alternate expression for TEM waves is
  $\mathbf{S}_{av} = \frac{\left|\tilde{\mathbf{E}}\right|^2}{2\eta}$ (no need to determine **H**)
- gives the power density per unit area of an EM wave (unit is the W/m^2)
- points in the direction of power flow and propagation direction of phase fronts (in most or all media that we are considering in this course)

Radiation pattern

- plot of $|\mathbf{S}_{av}|$ (sometimes normalized), directivity, or gain vs. θ and/or ϕ
- usually plotted using a dB (or dBi for directivity and gain) scale
- normalized power pattern:
  $F(\theta,\phi) = \frac{|\mathbf{S}_{av}|}{S_{\max}}$
- $\max\{F(\theta, \phi)\} = 1$
- interpretation of radiation pattern plot (either in terms of actual gain/directivity or the normalized power pattern)
- determination of relative power in various directions
- determination of half-power beamwidth

To find far fields of arbitrary current that flows along z-axis, use

$$d\tilde{\mathbf{E}} = \hat{\boldsymbol{\theta}} \frac{jk\eta I(z')dz'}{4\pi R} e^{-jkR'} \sin\theta, \quad d\tilde{\mathbf{H}} = \hat{\boldsymbol{\varphi}} \frac{jkI(z')dz'}{4\pi R} e^{-jkR'} \sin\theta$$

These are the far fields radiated by a Hertzian dipole of length dz, where primed coordinates refer to position z' along current distribution. Leads to

$$\tilde{\mathbf{E}} \approx \hat{\boldsymbol{\theta}} \frac{jk\eta}{4\pi R} e^{-jkR} \sin\theta \int_{-l/2}^{l/2} I(z') e^{jkz'\cos\theta} dz'$$

Far fields of short dipole (triangular current distrib.) of length l (peak units):

$$\tilde{\mathbf{E}} = \hat{\boldsymbol{\theta}} \frac{jk\eta \tilde{I}_{in} l}{8\pi R} e^{-jkR} \sin\theta, \quad \tilde{\mathbf{H}} = \hat{\boldsymbol{\varphi}} \frac{jk\tilde{I}_{in} l}{8\pi R} e^{-jkR} \sin\theta$$

Approximation of current distribution along center-fed dipoles:

- start with current distribution on an open-circuited parallel-wire transmission line stub
- bend transmission line wires near end of stub outward 90° (fold point is $l/2$ from end of stub) so that folded wires are collinear (in line with each other)
- short dipole has a nearly triangular current distribution because the "ends" of the sinusoidal distribution are nearly linear
- Hertzian dipole has uniform current distribution, which is not physically possible, but it is a useful concept. Uniform current can be approximated using "capacity hats" (charge reservoirs) at the outer ends of the wire halves, which allow the currents at the ends of the dipole to be non-zero.

Directivity and gain

- calculation of radiated power (equal to input power if no losses)

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} \mathbf{S}_{av}(\theta,\phi)\cdot\hat{\mathbf{R}}R^2 \sin\theta\, d\theta\, d\phi = S_{\max}R^2 \int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta\, d\theta\, d\phi$$

- concept of isotropic radiator
  - hypothetical antenna that radiates with equal intensity in all directions
  - radiated fields have no specific polarization (not realistic)
  - Poynting vector of isotropic radiator: $\mathbf{S}_{iso} = \hat{\mathbf{R}}\dfrac{P_{in}}{4\pi R^2}$, where P_{in} is input power to isotropic antenna, which is assumed to be lossless
  - used as a standard of comparison between actual antennas
- directivity calculated from normalized power pattern $F(\theta, \phi)$

$$D = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} F(\theta,\varphi)\sin\theta\, d\theta\, d\varphi}$$

- other relationships involving directivity

$$D = \frac{S_{\max}}{|\mathbf{S}_{iso}|} = 4\pi R^2 \frac{S_{\max}}{P_{rad}} = 4\pi R^2 \frac{S_{\max}}{P_{in}}$$ (2nd equality assumes **no power losses**)

- if there are power losses, then $G = \xi D$, where G is the gain and ξ is the efficiency. Also,

$$G = \frac{S_{\max}}{|\mathbf{S}_{iso}|} = 4\pi R^2 \frac{S_{\max}}{P_{in}} = 4\pi R^2 \xi \frac{S_{\max}}{P_{rad}}$$, because $P_{rad} = \xi P_{in} \quad \rightarrow \quad P_{in} = \dfrac{P_{rad}}{\xi}$

- gain & directivity are usually expressed in dBi (decibels relative to an isotropic radiator):

  $D[\text{dBi}] = 10\log(D)$ $\qquad D = 10^{D[\text{dBi}]/10} = 10^{0.1D[\text{dBi}]}$

  $G[\text{dBi}] = 10\log(G)$ $\qquad G = 10^{G[\text{dBi}]/10} = 10^{0.1G[\text{dBi}]}$

  Also note that

  $G = \xi D \;\rightarrow\; 10\log(G) = 10\log(\xi) + 10\log(D) \;\rightarrow\; G[\text{dBi}] = \xi[\text{dB}] + D[\text{dBi}]$

  where ξ, because it is less than 1, always has a negative value in dB
- dBi unit (gain/directivity in dB referenced to isotropic radiator) vs. dB unit
- directivities of short dipole, Hertzian dipole, and small loop are all 1.5 (1.76 dBi) because their normalized power patterns are all $\sin^2\theta$

Pattern solid angle

- definition and relationship to directivity:

$$\Omega_p = \int_0^{2\pi}\int_0^{\pi} F(\theta,\varphi)\sin\theta\, d\theta\, d\varphi \quad \text{and} \quad D = \frac{4\pi}{\Omega_p}$$

- approximation of Ω_p for highly directive antennas with one major radiation pattern lobe:

  $\Omega_p \approx \text{HPBW}_1 \cdot \text{HPBW}_2$,

  where HPBW_1 and HPBW_2 are the half-power beamwidths in the first and second principal pattern cuts expressed in radians

Radiation resistance

- input impedance of antenna: $Z_{in} = R_{rad} + R_{loss} + jX_{in}$,

  where R_{rad} = radiation resistance, R_{loss} = loss resistance, X_{in} = input reactance
- R_{rad} is real part of equivalent input impedance that represents radiated power; it accounts for power delivered by transmission line that is radiated by antenna

- definition: $R_{rad} = \dfrac{2P_{rad}}{\left|\tilde{I}_{in}\right|^2}$, if $\tilde{I}_{in}$ represents peak (not rms) input current at the feed point; however, $\tilde{I}_{in}$ might not be the peak value of the current distribution along the antenna
- short dipole: $R_{rad} = 20\pi^2 \left(\dfrac{l}{\lambda}\right)^2$, where l = length
- Hertzian dipole: $R_{rad} = 80\pi^2 \left(\dfrac{l}{\lambda}\right)^2$
- half-wave dipole: R_{rad} = 73 Ω (ideal half-wave dipole isolated in free space)
- quarter-wave monopole: R_{rad} = 36.5 Ω = 73/2 Ω (ideal monopole over perfectly conducting ground plane of infinite extent)

Gain (G) and efficiency (ξ)

- $G = \xi D$, where D is the directivity
- $P_{rad} = \xi P_{in}$
- $\xi = \dfrac{R_{rad}}{R_{rad} + R_{loss}}$, if R_{rad} and R_{loss} are in series in the input impedance model
- loss resistance usually represents finite conductivity of antenna structure and/or ground beneath it, but other factors can contribute to loss as well
- calculation of power density at a distance given gain or directivity and input power to antenna (does not include xmsn line, impedance matching network, and polarization mismatch losses):

$$S_{\max} = \left|\mathbf{S}_{av}\right|_{\max} = G\frac{P_{in}}{4\pi R^2} = \xi D \frac{P_{in}}{4\pi R^2}$$

Loss resistance R_{loss} due to finite conductivity of antenna

- real part of equivalent input impedance of antenna that accounts for power delivered by transmission line that is absorbed (not radiated) by antenna structure and perhaps by some surrounding objects and/or ground
- definition: $R_{loss} = \dfrac{2P_{loss}}{\left|\tilde{I}_{in}\right|^2}$, with $\tilde{I}_{in}$ in peak (not rms) units
- Hertzian dipole: $R_{loss} = \dfrac{l}{2\pi a}\sqrt{\dfrac{\pi f \mu_c}{\sigma_c}}$ (lower case "L" in numerator, not "1")

  short dipole: $R_{loss} = \dfrac{l}{6\pi a}\sqrt{\dfrac{\pi f \mu_c}{\sigma_c}}$

  half-wave dipole: $R_{loss} = \dfrac{\lambda}{8\pi a}\sqrt{\dfrac{\pi f \mu_c}{\sigma_c}}$

  arbitrary-length dipole: $R_{loss} = \dfrac{1}{2\pi a\left|I_{in}\right|^2}\sqrt{\dfrac{\pi f \mu_c}{\sigma_c}}\displaystyle\int_{-l/2}^{l/2}\left|\tilde{I}(z)\right|^2 dz$

  where l = length of wire; a = radius of wire (assumed constant along length); f = operating frequency; μ_c = permeability of wire (usually μ_0); σ_c = conductivity of wire

Specialized computational methods like the one used in EZNEC are required to find accurate current distributions along real antennas, but current distributions on center-fed dipoles of arbitrary length can be approximated by imagining the conductors of an open-circuited stub being folded outward $l/2$ from the end.

Half-wave dipole

- expression for far field if dipole is center fed, oriented along z-axis, and operating in free space:
  $$\tilde{\mathbf{E}} = \hat{\boldsymbol{\theta}}\, j60\tilde{I}_{in}\frac{e^{-jkR}}{R}\left[\frac{\cos\left(0.5\pi\cos\theta\right)}{\sin\theta}\right]$$
- directivity is 1.64 (2.15 dBi)
- radiation resistance is approx. 73 Ω
- input reactance behaves much like that of an open-circuited transmission line stub that is close to $\lambda/4$ in length; X_{in} is theoretically zero for exact $\lambda/2$ length of dipole, positive for $l > \lambda/2$, and negative for $l < \lambda/2$ (if l is not too far from $\lambda/2$)
- current distrib. on a real half-wave dipole is not exactly sinusoidal, so actual resonant length is slightly less than $\lambda/2$

<u>Relevant course material:</u>

HW:	#5 and #6
Reading:	Assignments from Feb. 27 through Mar. 26, including the supplemental readings
	"Radiation Power and Directivity of Antennas"
	"Radiation Resistance, Efficiency, and Gain of Antennas"
	"Loss Resistance Calculations for Arbitrary Current Distributions"

This exam will focus primarily on the course outcomes listed below and related topics:

3. Relate the power density of an electromagnetic wave radiated in any direction to an antenna's gain, radiation pattern, and applied input power.

The course outcomes are listed on the Course Policies and Information sheet, which was distributed at the beginning of the semester and is available on the Syllabus and Policies page at the course web site. The outcomes are also listed on the Course Description page. Note, however, that some topics not directly related to the course outcomes could be covered on the exam as well.