

Homework Assignment #4 – due via Moodle at 11:59 pm on Monday, Mar. 2, 2026

Instructions, notes, and hints:

Provide the details of all solutions, including important intermediate steps. You will not receive credit if you do not show your work.

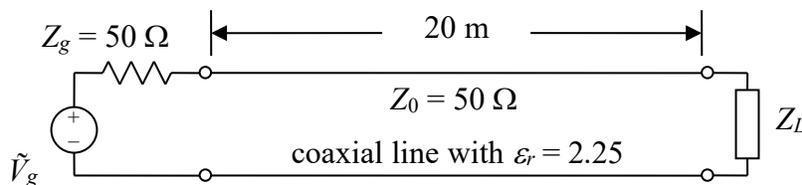
You may make reasonable assumptions and approximations to compensate for missing information, if any. In those cases, your answers could differ significantly from the posted answers. If you justify any approximations that you make, you will be given full credit.

The constitutive parameters (ϵ , μ , and σ) of many important engineering materials are available in Appendix B of the textbook (Ulaby and Ravaioli, 8th ed.).

Note that the first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

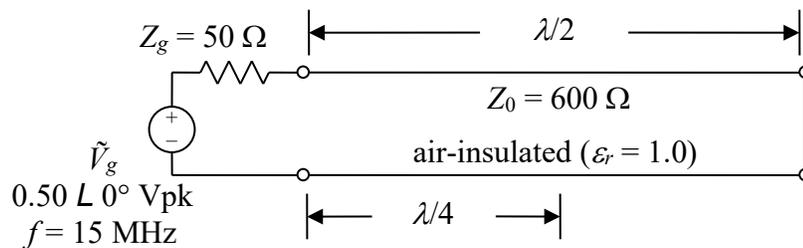
Graded Problems:

1. An antenna with an equivalent input impedance of $250 + j150 \Omega$ is to be connected to a parallel-wire air-insulated transmission line with $Z_0 = 450 \Omega$. The operating frequency is 20 MHz. The designers would like to achieve an impedance match by inserting an inductor or capacitor in parallel with the line at an appropriate distance from the load. Determine the location nearest to the load at which each type of component should be inserted. Also specify the required inductance and capacitance values.
2. Suppose in the previous problem that a shorted stub will be used at each location instead of a lumped reactance. Find the required physical length of each stub. The stubs are to be made from sections of 450Ω parallel-wire transmission line with air insulation.
3. In the system shown below with an unknown load impedance Z_L , a power meter inserted in a coaxial transmission line indicates that the incident power flowing toward the load is $P_{av}^i = 45 \text{ W}$ and the reflected power from the load is $P_{av}^r = 5.5 \text{ W}$. The line has polyethylene insulation with $\epsilon_r = 2.25$. The wavelength within the dielectric is 8.0 m, and the operating frequency is 25 MHz. Find the VSWR along the line.



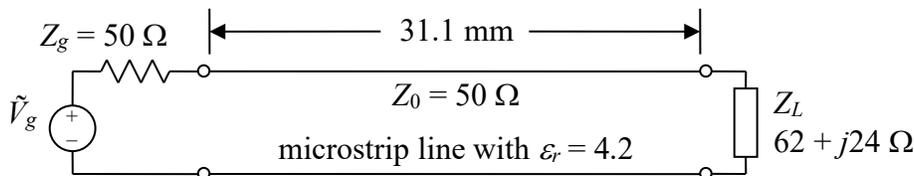
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4. As shown below, a $600\ \Omega$ air-insulated parallel-wire transmission line is connected to a function generator set to $0.50\ \text{V}_{\text{pk}}$ (purely real) at a frequency of $15\ \text{MHz}$. The line is exactly $\lambda/2$ long, and the end opposite the function generator is shorted. Find the total line voltage at the midpoint of the transmission line ($\lambda/4$ away from the function generator). Also find the forward and reflected power flowing along the line. The input impedance of the line is zero.



5. Depicted in the diagram below is a WLAN (wireless local-area network) signal source with an output impedance Z_g of $50\ \Omega$ (purely real) that can supply $5.0\ \text{mW}$ of power to a load under matched conditions (i.e., when a $50\ \Omega$ load is connected to the source). An amplifier with an input impedance of $62 + j24\ \Omega$ is connected to the signal source through a $50\ \Omega$ microstrip transmission line with a length of 1.2λ . The line has an effective relative permittivity of 4.2 , and the operating frequency range has a narrow bandwidth centered at $5.65\ \text{GHz}$. Assuming that the line is lossless, find:
- the time-average incident power that flows along the microstrip line.
 - the time-average reflected power that flows along the microstrip line.
 - the time-average real power and reactive power delivered by the signal source (represented by V_g and Z_g) to the input end of the microstrip line.
 - the time-average real power and reactive power delivered to the mismatched input port of the amplifier represented by Z_L .
 - the reactive power absorbed or delivered by the microstrip line itself using the expression below, where V_0^+ is in peak units. Make sure that the reactive power in the system balances, keeping in mind that there could be some round-off error.

$$Q = \frac{|\Gamma| |V_0^+|^2}{Z_0} [\sin(\theta_r - 2\beta l) - \sin \theta_r]$$



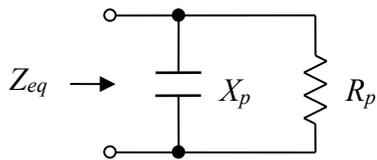
6. Suppose that in the previous problem a series capacitor is inserted into the microstrip line to match the amplifier's input impedance to the $50\text{-}\Omega$ line. The capacitor has a value of $1.2\ \text{pF}$ and is located $12\ \text{mm}$ (0.465λ) from the load. How much power is delivered to the load in this case? You might be able to solve this problem with minimal effort, but you must provide some kind of justification for your answer.

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Ungraded Problems:

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then come back to the difficult one after a few hours.

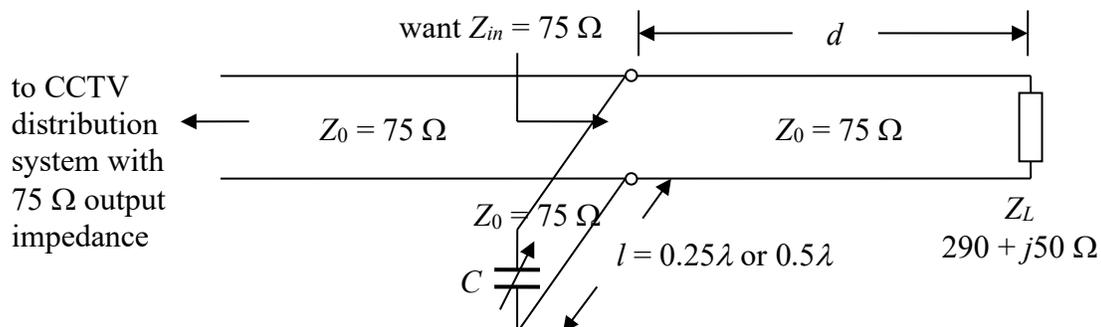
- The formula for l_{main} shown below is used in the design of shunt element-based matching networks. Suppose that a network is to be designed to match a load impedance of $19.5 - j24.4 \Omega$ to a 75Ω transmission line. At the distance l_{main} from the load, the equivalent input impedance can be modeled by the parallel equivalent circuit shown below (represented by R_p and X_p ; the p subscripts are for “parallel”). Find the values of R_p and X_p that constitute an appropriate model for this case. Note that Z_{eq} is the impedance seen looking into the line toward the load at the distance l_{main} from the load. The dimensions of the equivalent circuit below are tiny compared to a wavelength.



$$l_{main} = \frac{\lambda}{4\pi} [\theta_r \pm \cos^{-1}(-|\Gamma|)]$$

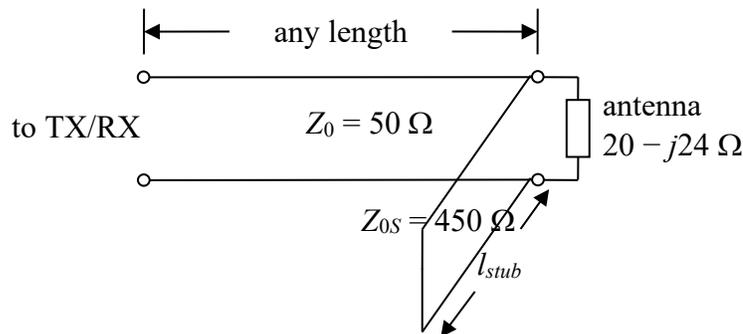
- A day-care center wants to install a closed-circuit television (CCTV) system to monitor the facility using old donated TV sets, which will all be tuned to old NTSC channel 3 (center frequency of 69 MHz). Rough measurements reveal that the TV sets have input impedances of roughly $290 + j50 \Omega$. The CCTV distribution system uses 75Ω coaxial cable with polyethylene insulation, so a single shunt-stub matching system made from the same cable will be added to each TV set. A “T” connector will be used to connect the stub in parallel with the main transmission line. To allow for the unknown input impedances of the TV sets, the length of each stub will be either $\lambda/4$ or $\lambda/2$, and a variable capacitor will be mounted at the end of each stub to provide adjustability. The matching system is depicted below.

- Find the shortest distance d from the load at which the stub should be placed.
- Given that a variable capacitor terminates each stub, determine whether its length should be $\lambda/4$ or $\lambda/2$. Think about how the capacitive load impedance would be transformed in each case. Find the physical length of the selected stub in cm.
- Find the center (average) value of the variable capacitor. Variable capacitors have maximum and minimum values, of course, so an actual system would be designed so that the average expected required capacitance would fall in the middle of the range. That should allow enough adjustability to achieve a good – but maybe not perfect – impedance match regardless of the actual load impedance.



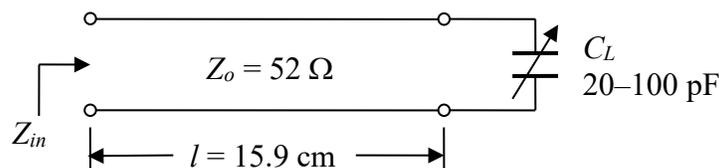
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3. A class of antennas known as Yagi-Uda arrays (after the Japanese engineers who invented the concept in the 1920s) typically have low input impedances relative to commonly used $50\ \Omega$ coaxial cable. One way to achieve an impedance match is to adjust one of the parts of the antenna so that the input impedance is complex with a negative imaginary part (capacitive). Then an inductive stub is connected in parallel with the antenna's terminals, and the result is an excellent match. Depicted below schematically is the feedline and input impedance of a Yagi-Uda array that has already been adjusted to have an appropriate capacitive input impedance. Find the required length of a shorted stub that would result in a good match to $50\ \Omega$. Note that the characteristic impedance Z_{0S} ("S" for stub) of the line section that makes up the stub is $450\ \Omega$, not $50\ \Omega$; it is made from a short piece of parallel-wire line rather than coax. The frequency of operation is $144\ \text{MHz}$ (the amateur radio 2-meter band). This type of impedance matching system is called a "hairpin" match; many examples can be found online by searching the term "hairpin match yagi." *Hint:* The required input susceptance of the stub is *not* $1/24 = 0.0417\ \text{S}$.



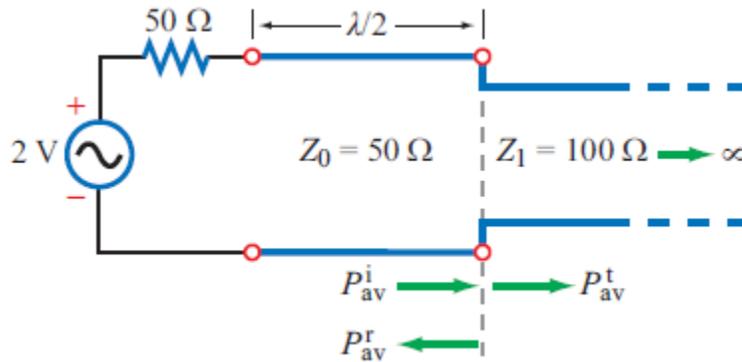
4. As shown below, a $20\text{--}100\ \text{pF}$ variable capacitor is attached to the end of a $15.9\ \text{cm}$ long section of RG-58A coaxial transmission line ($Z_0 = 52\ \Omega$ with polyethylene insulation). The operating frequency is $90.5\ \text{MHz}$, which is the assigned frequency for Bucknell's FM radio station WVBU.
- Find the input impedance Z_{in} of the line for the minimum value of C_L ($20\ \text{pF}$).
 - Find the input impedance when C_L is at the high end of its range ($100\ \text{pF}$).
 - If possible, find the value of C_L (in pF) at which the line's input impedance Z_{in} is zero at $90.5\ \text{MHz}$.
 - For the capacitance found in part c, find the input impedance of the line at $10\ \text{kHz}$.

The loaded stub considered in this problem is sometimes called a "wave trap." It is an effective way to keep a troublesome signal at a specific high frequency out of test equipment while minimizing the effect on low-frequency signals. The wave trap shown below would be placed in parallel with the test leads at the input port of a piece of test equipment (like an oscilloscope).



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5. A signal source with an output impedance Z_g of $50\ \Omega$ (purely real) and that has an open-circuit output voltage (i.e., Thévenin equivalent voltage) of $2.0\ \text{V}_{\text{pk}}$ (purely real drives a half-wavelength long $50\ \Omega$ line. Connected to the $50\ \Omega$ line at its load end (i.e., opposite the signal source) is a very long and slightly lossy $100\ \Omega$ line. The $100\ \Omega$ line is so long that any waves reflected at its far end are so attenuated by the time they return to the junction with the $50\ \Omega$ line that they are negligibly weak. The configuration is depicted below (Fig. P2.44 in the textbook, Ulaby and Ravaioli, 8th ed.). Find the time-average incident and reflected power P_{av}^i and P_{av}^r along the $50\ \Omega$ line. Also find the power P_{av}^t that is delivered (transmitted, hence the “t” superscript) to the input end of the $100\ \Omega$ line. You may assume that the $50\ \Omega$ line is lossless.



6. The $1.0\ \text{mV}_{\text{pk}}$ (purely real) ideal voltage source and impedance Z_g in the diagram below represent a signal source. Find the maximum available power P_A that the signal source can supply to a load. The three resistors to the right of the source form a $20\ \text{dB}$ T-network attenuator. The resistors have tight tolerances (1%), so their values are indicated with three digits of precision. The attenuator is designed to preserve the $50\ \Omega$ output impedance of the signal source connected to it. That is, the Thévenin equivalent resistance of the complete circuit to the left of terminals $a-b$ is $50\ \Omega$. (Verify this for yourself.) Find the available power of the signal source with the attenuator circuit attached, and compare it to the available power without the attenuator. Express the difference between the two power levels in decibels. *Hint:* This is a $20\ \text{dB}$ attenuator.

