ECEG 390 Theory and Applications of Electromagnetics Spring 2026

Homework Assignment #7 – due via Moodle at 11:59 pm on Monday, Apr. 13, 2026

***Instructions, notes, and hints*:**

Provide the details of all solutions, including important intermediate steps. You may make reasonable assumptions and approximations to compensate for missing information, if any. If your answers differ from the posted answers but you justify any approximations that you make, you will be given full credit.

Note that the first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems.

***Graded Problems*:**

1. The WVBU transmitter on the roof of the Dana Engineering Building has an output power of around 400 W, which is supplied to an antenna with a gain of around 6.0 dBi. Assume that the transmission line between the transmitter and the antenna has 2.0 dB of loss and that all receivers are in the direction of maximum radiation from the transmit antenna. Suppose that a receiver tuned to WVBU at 90.5 MHz requires a received signal strength of –100 dBm (10^{-13} W) to produce mostly noise-free sound. The receiver uses a half-wave dipole antenna oriented so that the signal arrives from a direction that is within the half-power beamwidth of the antenna. Find the maximum distance between the receiver and the WVBU transmitting site that allows mostly noise-free reception. Assume a direct line-of-sight path between the transmit and receive antennas, and neglect reflections from the ground and nearby objects.

2. **[Deferred from HW #6 and modified]** A highly advanced circular reflector antenna has been fabricated using a new conductive polymer. The physical aperture is 67 cm in diameter, but because of "spillover" from the feed antenna, the aperture efficiency is only 60%. Although the antenna presents a 50 Ω load to the 50 Ω transmission line feeding it, measurements indicate that 10 Ω of the 50 Ω total is due to power loss from the limited conductivity of the polymer, the type of feed arrangement used, and various losses in the feed antenna. (Note that aperture efficiency and power efficiency are different effects and can exist simultaneously.) The antenna is to be used in an 8.0 GHz communication link between two identical facilities that are 20 km apart. Each facility has one of the new antennas and a transmitter that produces 5.0 W of output power; however, the transmission lines that connect the transmitters and receivers to the antennas each have 6.0 dB of loss. (The antenna can be switched between transmission and reception mode.) Each antenna is in the other's direction of maximum radiation.

   a. Find the power density in nW/m^2 in the vicinity of the antenna at one of the facilities when the other one is transmitting. Neglect atmospheric attenuation and reflections from the ground and other objects between the facilities.

   b. Find the signal power that appears at the input of the receiver when one of the facilities is being used in reception mode.

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3. A satellite ground station consists of a 1.0 kW transmitter and an antenna with a directivity of 26 dBi and an efficiency of 50%. The satellite's receive antenna has a directivity of 20 dBi and an efficiency of 90%. Find the variation in the signal power detected by the satellite's receiver as its orbit causes its distance to the ground station to vary from 200 km to 3,000 km. Neglect losses in the atmosphere, losses in the interconnecting transmission lines, and reflections from the ground, and assume that the ground station antenna tracks the satellite so that it is always in the direction of maximum gain. The frequency of operation is 12 GHz.

4. In a semi-rural area like Lewisburg, the noise levels in the AM broadcast band are such that a receiver detects roughly 0.5 nW of noise when tuned to a particular station. An incoming AM signal must be at least 10 dB above the noise level to be considered listenable by most people. The WMLK-AM transmitter operates at a frequency of 1070 kHz and is approximately 10 km from Lewisburg. It uses a monopole antenna with a gain of about 5.0 dBi. A receiver that is tuned to the signal has a 1.0 m-long center-fed short dipole antenna (triangular current distribution) made from an aluminum rod with a diameter of 4.8 mm. Find the minimum transmitter power required to maintain a listenable signal at the receiver. Assume a direct path between the receiving antenna and the transmitting antenna, neglect reflections from the ground and nearby objects, and assume that the transmission line and impedance matching network losses are negligible.

5. The receiver of a garage door opener (the part inside the garage) uses a circular loop antenna with a diameter of 2.0 cm. The opener operates at a frequency of 915 MHz. At the input terminals of the loop, a voltage with a peak value of at least 100 μV must be developed if the door is to open.

- **a.** If the loop is oriented for maximum response (i.e., maximum voltage across the terminals), find the peak magnitude of the magnetic field **H** in the vicinity of the loop.
- **b.** The loop is now tilted 60° away from the position of maximum response. For the same magnetic field as in part **a**, find the voltage that would be developed across the gap with the new orientation.

6. An engineering student builds a sensitive heart rate detector circuit on a piece of single-sided printed circuit board. (That is, there are copper traces on one side and no copper anywhere on the other side.) The copper circuit board traces from the sensor leading to the first stage of amplification are parallel, 4.0 cm long, and spaced 1.0 cm apart. The sensor, the copper traces, and the input terminals of the amplifier form a circuit loop. The sensor system is depicted in the diagram at the top of the next page. A nearby FM radio station operating at 100 MHz generates an electric field **E** that has a peak magnitude of 40 mV/m in the vicinity of the detector. Find the peak voltage at 100 MHz that is developed across the input of the amplifier by the FM signal when the PC board is oriented for maximum response to the signal. Assume that the sensor has an equivalent impedance of nearly zero at 100 MHz. Recall that the electric field and magnetic field magnitudes of the transverse electromagnetic (TEM) fields radiated by all antennas have the relationship $|\mathbf{E}|/|\mathbf{H}| = \eta$, where η is the intrinsic impedance of the medium in which the wave propagates. For air and free space, $\eta = 120\pi\,\Omega = 377\,\Omega$.

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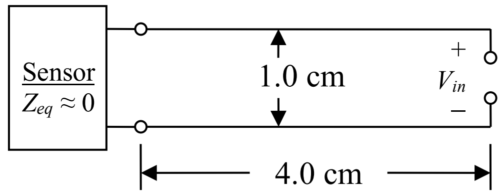


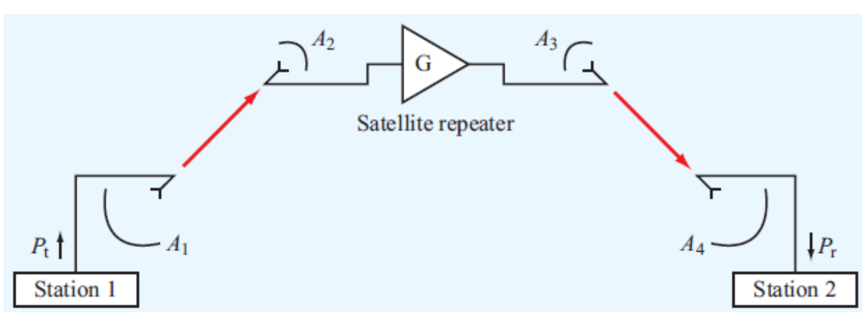
Diagram for Graded Prob. 6

7. The engineering student in the previous problem realizes that the FM radio station is being picked up by the heart rate detector's input circuitry and is interfering with its proper operation. The student redesigns the PC board by reducing the separation between the traces coming from the sensor from 1.0 cm to 1.0 mm. Find the largest possible peak voltage at 100 MHz that will be developed across the input of the amplifier with the new design. Assume that the width of the circuit board traces is negligible compared to the spacing between the traces.

***Ungraded Problems*:**

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one of them. Move on to something else and then come back to it after a few hours.

1. [adapted from Prob. 9.29 of Ulaby and Ravaoli, 7th ed.] The figure below depicts a satellite repeater with two antennas, one pointed toward ground station 1 and the other toward ground station 2. All antennas are parabolic dishes, and each one has an aperture efficiency of 60%. The transmission line between each ground station and its respective antenna has a loss of 3 dB. The line loss between the satellite antennas and the repeater is negligibly small. Antennas A_1 and A_4 are each 4.0 m in diameter, and antennas A_2 and A_3 are each 2.0 m in diameter. The distance between the satellite and each of the ground stations is 40,000 km. The satellite transponder receives the signal from station 1 using antenna A_2, amplifies the signal by 80 dB, and then uses antenna A_3 to retransmit the signal to ground station 2. The system operates at 10 GHz with P_t = 1.0 kW (transmitter power at station 1). Determine the received power P_r at station 2.



[Figure source: F. T. Ulaby and U. Ravaioli, Fundamentals of Applied Electromagnetics, 7th ed., Pearson Education, Inc., Upper Saddle River, NJ, 2025.]

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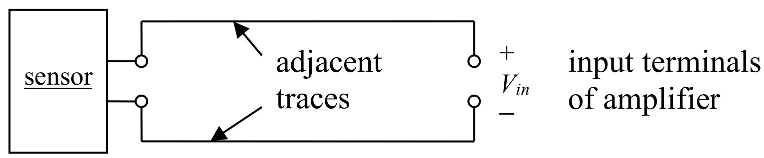
2. The interference from the FM station considered in two of the graded problems can be reduced even further by using a twisted pair of wires from the sensor to the amplifier's input terminals. A comparison of the two feed arrangements is shown below. In the upper diagram, the leads from the sensor to the amplifier are straight and parallel. In the lower diagram, the leads are twisted. (You may assume that the wires in the twisted pair form either flat loops or helices; the answer is the same either way.) Provide a brief qualitative explanation based on Faraday's law for why a twisted pair usually has better signal rejection than two parallel wires; that is, explain why the voltage induced at the terminals when a twisted pair is used is much less than that induced when parallel wires are used.



H-field vector points into/out of page.