ECEG 390 Theory and Applications of Electromagnetics Spring 2026

Homework Assignment #9 – due via Moodle at 11:59 pm on Monday, May 4, 2026

***Instructions, notes, and hints*:**

Provide the details of all solutions, including important intermediate steps. You may make reasonable assumptions and approximations to compensate for missing information, if any. If your answers differ from the posted answers but you justify any approximations that you make, you will be given full credit.

Unless otherwise indicated, you may use Matlab, Mathematica, or other software to make difficult or time-consuming calculations. If you do, include a copy of the file or screen display that shows your work.

The first set of problems will be graded and the rest will not be graded. Only the graded problems must be submitted by the deadline above. Do not submit the ungraded problems. **You are responsible for understanding the concepts that are the focus of ungraded problems.**

***Graded Problems*:**

1. A two-way radio link operating at 75 MHz has identical stations at each end. Each one has a transmitter that produces an output power of 100 W, a vertical half-wave dipole antenna (oriented parallel to the *z*-axis), and a transmission line that connects the transmitter/receiver to the antenna and that has a loss of 3.5 dB. The two stations are on mountain tops and are 7.2 km apart. Due to reflections from the intervening valley and from nearby objects, the electric field at the reception site can be expressed as shown below. The transmitter is at the origin of the coordinate system. Assume that each antenna is oriented so that the other antenna is in the direction of maximum radiation, that both antennas are essentially 100% efficient, and that there are no impedance mismatch or matching network losses.

- **a.** Find the normalized incident electric field $\hat{\mathbf{e}}_{TX}$ in the vicinity of the receiver site.
- **b.** Find the received signal power P_{RX}. Remember to include the polarization loss factor (PLF).

$$\tilde{\mathbf{E}} = \hat{\boldsymbol{\varphi}}\, 14.4 e^{-j0.7\pi} \frac{e^{-j1.57R}}{R} + \hat{\boldsymbol{\theta}}\, 64.8 e^{-j0.8\pi} \frac{e^{-j1.57R}}{R} \text{ V/m}$$

2. Find the polarization loss factor (PLF) between the incident wave given in the previous problem and an antenna with right-hand circular polarization (RHCP).

3. The expression below describes a LHEP wave with an axial ratio close to 1 (0 dB); that is, the tip of the E-field arrow traces out an ellipse that is almost circular. Suppose that the wave is being received by a RHCP antenna. Show that the polarization loss factor is small but not zero. Express the PLF in decibels.

$$\tilde{\mathbf{E}} = \hat{\mathbf{x}}\, 0.70 e^{-j0.188z} + \hat{\mathbf{y}}\, 0.70 e^{j0.44\pi} e^{-j0.188z} \text{ mV/m}$$

(*continued on next page*)

4. Using a series of measurements, a team of engineers finds that a certain nonmagnetic material (i.e., with $\mu = \mu_0$) is characterized at a frequency of 1.0 GHz by $\alpha = 0.030$ Np/m and $\beta = 74$ rad/m. Find the values of the constitutive parameters ε and σ of the material. Also find the speed of a wave that propagates through the material. If you assume that the material is either a good conductor or a low-loss dielectric, you must verify your assumption; however, no assumptions are necessary to solve the problem. *Hint*: $\gamma = \alpha + j\beta = j\omega\sqrt{\mu\varepsilon_c}$

5. [adapted from Prob. 7.42 of Ulaby & Ravaioli, 7th ed.] A team of scientists is designing a radar to measure the thickness of an Antarctic ice shelf. If the echo due to the reflection from the ice-rock boundary is to be detectable, the thickness of the ice sheet should not exceed three skin depths. Assuming that $\varepsilon'_r = 3.0$ and $\varepsilon''_r = 0.010$ for the ice shelf and that the maximum anticipated ice thickness in the area under exploration is 1.2 km, find the usable frequency range of the radar.

6. The attenuation in air at sea level for signals at 30 GHz is approximately 0.14 dB/km. This frequency is close to one of many being considered for future 5G cellular systems. Suppose that the electric field component of a particular plane wave at this frequency propagating through air at sea level is expressed mathematically (and partially symbolically) as

$$\tilde{\mathbf{E}} = \hat{\mathbf{x}}\frac{37}{z}e^{-\alpha z}e^{-j\beta z} \text{ mV/m.}$$

a. Find the attenuation constant α in Np/m.
b. Find the phase constant β in rad/m.
c. Find the numerical values of the power density at $z = 5.0$ m (close to the origin of the wave) and at $z = 5.0$ km with and without the path loss due to the attenuation constant α included. You may assume that air is a low-loss dielectric.
d. Express the difference in the results obtained in part **c** for the distance $z = 5.0$ km in the decibel unit.

***Ungraded Problems*:**

The following problems will not be graded, but you should attempt to solve them on your own and then check the solutions. Do not give up too quickly if you struggle with one or more of them. Move on to a different problem and then come back to the difficult one after a few hours.

1. Find the skin depth of copper wire at the following frequencies, and compare the results to the radius of #20 AWG copper wire:

a. 60 Hz
b. 1.0 MHz (middle of AM broadcast band)
c. 100 MHz (middle of FM broadcast band)
d. 2.5 GHz (within the lowest Wi-Fi frequency range)

(*continued on next page*)

2. The constitutive parameters of soil are highly variable and depend on such attributes as moisture content, mineral content, and density. The antenna analysis software *EZNEC* uses the values $\varepsilon_r = 15$, $\mu_r = 1$, and $\sigma = 0.005$ S/m to represent "average" ground. Classify "average" ground as a good conductor, a quasi-conductor, or a low-loss dielectric at each of the following frequencies. For each case, also calculate the wavelength and the skin depth. List the results in tabular form at the end for convenience.

- **a.** 60 Hz (power line frequency in the US and many other countries)
- **b.** 60 kHz (operating frequency of WWVB, the NIST LF time standard station)
- **c.** 1070 kHz (operating frequency of WKOK)
- **d.** 90.5 MHz (operating frequency of WVBU)
- **e.** 2.44 GHz (middle of the Bluetooth frequency range)

3. The electric field component of a planar TEM wave traveling in a nonmagnetic medium with $\varepsilon_r = 9.0$ can be represented by the expression below. Determine the propagation direction of the wave and the time-average power density carried by the wave.

$$\tilde{\mathbf{E}} = \hat{\mathbf{y}}16.0e^{-j0.2\pi}e^{j0.157x} + \hat{\mathbf{z}}17.0e^{-j0.7\pi}e^{j0.157x} \text{ mV/m}$$

4. A plane wave at 400 MHz propagates in a lossy dielectric with constitutive parameters $\varepsilon = 8.0\varepsilon_0$, $\mu = \mu_0$, and $\sigma = 0.38$ S/m. Find the phase difference between the electric field and the magnetic field. *Hint*: If the medium were lossless, the **E** and **H** fields would be in phase.

5. For a lossy medium, the propagation constant γ is related to the medium's constitutive parameters as shown below. Also shown below are the two equations that provide the starting point for deriving the expressions for α and β, the real and imaginary parts, respectively, of γ. Show how the formulas for α and β are derived.

$$\gamma^2 = (\alpha + j\beta)^2 = -\omega^2\mu(\varepsilon' - j\varepsilon'') \rightarrow \alpha^2 - \beta^2 = -\omega^2\mu\varepsilon' \text{ and } 2\alpha\beta = \omega^2\mu\varepsilon''$$

$$\alpha = \omega\left\{\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1\right]\right\}^{1/2} \quad \text{and} \quad \beta = \omega\left\{\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} + 1\right]\right\}^{1/2}$$