ECEG 470/670 Communication and Information Systems Fall 2025

Policies and Review Topics for Exam #3

The following policies will be in effect for the exam. They will be included in a list of instructions and policies on the first page of the exam:

1. You will be allowed to use a non-wireless enabled calculator, such as a TI-89.
2. You will be allowed to use three 8.5 × 11-inch two-sided handwritten help sheets. No photocopied material or copied and pasted text or images are allowed. If there is a table or image from the textbook or some other source that you feel would be helpful during the exam, please notify me.
3. All help sheets will be collected at the end of the exam but will be returned to you either immediately or soon after the exam.
4. Use of a help sheet that is not completely handwritten will result in an automatic 5-point score reduction.
5. If you begin the exam after the start time, you must complete it in the remaining allotted time. However, you may not take the exam if you arrive after the first student has completed it and left the room. The latter case is equivalent to missing the exam.
6. **You may not leave the exam room without prior permission except in an emergency or for an urgent medical condition. Please use the restroom before the exam.**

The exam will begin at 2:00 pm on Thursday, November 20 in Dana 117. You will have until 3:50 pm to complete the exam.

The following is a list of topics that could appear in one form or another on the exam. Not all of these topics will be covered, and it is possible that an exam problem could cover a detail not specifically listed here. However, this list has been made as comprehensive as possible. You should be familiar with the topics on the previous review sheet in addition to those listed below.

Although significant effort has been made to ensure that there are no errors in this review sheet, some might nevertheless appear. The textbook and the supplemental readings are the final authority in all factual matters, unless errors have been specifically identified there. You are ultimately responsible for obtaining accurate information when preparing for your exam.

Pulse code modulation (PCM)

- quantization noise in PCM
  - quantization error lies in interval $(-0.5\Delta v, 0.5\Delta v)$, where $\Delta v = \dfrac{2m_p}{L}$
  - mean squared quantization error: $\overline{q^2(t)} = \dfrac{m_p^2}{3L^2}$
  - signal-to-quantization-noise ratio (SQNR): $\text{SQNR} = \dfrac{S_o}{N_o} = 3L^2 \dfrac{\overline{m^2(t)}}{m_p^2}$

  - mean square message signal amplitude (related to power level):
    $$\overline{m^2(t)} = \lim_{T\to\infty} \frac{1}{T}\int_t^{t+T} m^2(\tau)\,d\tau$$
  - for a sinusoid with magnitude m_p, $\overline{m^2(t)} = \frac{m_p^2}{2}$.

    Note that this is the mean squared amplitude averaged over a complete cycle of amplitude values; if divided by the applicable resistance, it is equal to the power in the signal. The mean squared *peak* amplitude for such a signal is m_p^2.
- companding in PCM
  - μ-law compression (used mainly in North America and Japan)
    $$y = \frac{1}{\ln(1+\mu)} \ln\left(1 + \mu\frac{|m|}{m_p}\right), \quad 0 \le \frac{|m|}{m_p} \le 1$$
    for positive m; multiply by -1 for negative m, where m = message sig. amplitude and y = output amplitude of compressor stage (normalized to 1)
  - A-law compression (used mainly in rest of world and on international routes)
    $$y = \begin{cases} \dfrac{A}{1+\ln A}\left(\dfrac{|m|}{m_p}\right), & 0 \le \dfrac{|m|}{m_p} \le \dfrac{1}{A} \\ \dfrac{1}{1+\ln A}\left(1 + \ln\dfrac{A|m|}{m_p}\right), & \dfrac{1}{A} \le \dfrac{|m|}{m_p} \le 1 \end{cases}$$
    for positive m; multiply by -1 for negative m, where m = message sig. amplitude and y = output amplitude of compressor stage (normalized to 1)
- min. channel bandwidth of PCM signals:
  - $B_T = nB$, if sampling rate is exactly equal to Nyquist limit
    or $B_T = 0.5nf_s$, where f_s = sampling rate
  - in practice, $f_s > 2B$, so actual bandwidth of PCM signal is greater than B_T
  - bandwidth-to-SNR trade-off: For each bit added to PCM signal, bandwidth increases proportionately but SNR improves by 6 dB

Differential PCM (DPCM)
- transmits the difference between the actual current message signal sample value $m[k]$ and the predicted current message signal sample value $\hat{m}[k]$, where k = current sample no.
- quantization range of difference signal expected to be smaller than quantization range of full message signal; thus, quantization noise for DPCM should be much smaller than with regular PCM for the same number of quantization levels L or transmitted bits n ($L = 2^n$); alternatively, the number of bits n can be reduced while achieving the same SNR
- wide range of prediction algorithms are available; many are based on finite difference approximations to 1st, 2nd, 3rd, etc. derivatives; can also used a tapped delay line with weights determined by minimum squared error (MSE) algorithm
- receiver (RX) uses the same prediction algorithm as transmitter (TX), so RX generates the same predicted value of $m[k]$ as the TX
- actual difference signal: $d[k] = m[k] - \hat{m}[k]$
- quantized difference signal: $d_q[k] = d[k] + q[k]$, where $q[k]$ = quantization noise
- resulting quantized message signal: $m_q[k] = m[k] + q[k]$

- improvement in SNR: $G_p = \left(\dfrac{m_p}{d_p} \right)^2$, where d_p = peak amplitude of quantizer for $d(t)$ and m_p = peak amplitude of quantizer for $m(t)$
- in adaptive DPCM, the value of d_p is changed if past values of $d_q[k]$ have been very large or very small; TX and the RX follow the same adjustment rules

Delta modulation

- essentially one-bit DPCM but with much higher sampling rate (at least 4× what would be used in DPCM)
- assumption is that with higher sampling rate, the difference between adjacent samples is much smaller than what would be obtained with DPCM at a rate just above the Nyquist limit
- major advantage: binary word framing is unnecessary if only one bit per sample is sent; there is much less overhead, so the expense of oversampling is more than compensated by sending only one bit per sample
- DM uses a first-order predictor so that $m_q[k] = m_q[k-1] + d_q[k]$
- quantized message signal at any given sample point is simply the accumulation of quantized differences up to that point, assuming that $m_q[0] = 0$:

  $$m_q[k] = \sum_{m=1}^{k} d_q[m]$$
- the "delta" signal $d_q[k]$ is essentially the slope of $m(t)$ at the corresponding instant in time; the slope can be positive or negative; that is how delta modulation got its name
- threshold of coding
  - if variation in amplitude of $m(t)$ is smaller than the DM encoding slope interval E for long periods, then the output of the DM modulator oscillates between $\pm E$ (idling); this leads to granular noise
  - granular noise level: $N_o = \dfrac{E^2 B}{3 f_s}$,

    where B = message signal bandwidth and f_s = sampling frequency (Nyquist limit is $f_{Ny} = 2B$, but fs >> f_{Ny} for DM)
  - granular noise is reduced the more oversampled the signal is
- slope overloading in DM
  - occurs when $m(t)$ changes more than E during a sample interval
  - amplitude step size should be limited to

    $$\left|\dot{m}(t)\right|_{\max} < \frac{E}{T_s} = Ef_s \quad \rightarrow \quad E > \frac{\left|\dot{m}(t)\right|_{\max}}{f_s}$$
  - special case of sinusoidal message signal: $m(t) = A\cos\omega t$

    $$\left|\dot{m}(t)\right|_{\max} < \omega A \quad \rightarrow \quad \omega A > Ef_s \quad \rightarrow \quad A_{\max} < \frac{Ef_s}{\omega}$$
  - voice signals typically have significant frequency content up to 4 kHz, but using that frequency to set the value of E is too conservative because spectral density decays as frequency approaches 4 kHz
  - rule-of-thumb for voice signals: calculate $A_{\max}$ using $\omega = 2\pi(800\text{ Hz})$

<u>Relevant course material:</u>

Homework: #5
Mini-Projects: #2 and #3
Reading: Assignments from Oct. 27 through Nov. 7

This exam will focus primarily on the course outcomes listed below and related topics.

4. Demonstrate how pulse code modulation systems encode analog signals into digital form.
5. Relate signal-to-quantization noise ratio to various methods for quantizing and compressing pulse code modulated signals.

The course outcomes are listed on the Course Policies and Information sheet, which was distributed at the beginning of the semester and is available on the Syllabus and Policies page at the course web site. The outcomes are also listed on the Course Description page. Note, however, that some topics not directly related to the course outcomes could be covered on the exam as well.