ECEG 470/670 Communication and Information Systems Fall 2025

Policies and Review Topics for Final Exam

The following policies will be in effect for the exam. They will be included in a list of instructions and policies on the first page of the exam:

1. You will be allowed to use a non-wireless enabled calculator, such as a TI-89.
2. You will be allowed to use four 8.5 × 11-inch two-sided handwritten help sheets. No photocopied material or copied and pasted text or images are allowed. If there is a table or image from the textbook or some other source that you feel would be helpful during the exam, please notify me.
3. All help sheets will be collected at the end of the exam but will be returned to you either immediately or soon after the exam.
4. Use of a help sheet that is not completely handwritten will result in an automatic 5-point score reduction.
5. If you begin the exam after the start time, you must complete it in the remaining allotted time. However, you may not take the exam if you arrive after the first student has completed it and left the room. The latter case is equivalent to missing the exam.
6. **You may not leave the exam room without prior permission except in an emergency or for an urgent medical condition. Please use the restroom before the exam.**

The final exam will take place 8:00–10:50 am on Thursday, December 18 in Dana 117.

The following is a list of topics that could appear in one form or another on the exam. Not all of these topics will be covered, and it is possible that an exam problem could cover a detail not specifically listed here. However, this list has been made as comprehensive as possible. You should be familiar with the topics on the previous review sheet in addition to those listed below.

Although significant effort has been made to ensure that there are no errors in this review sheet, some might nevertheless appear. The textbook and the supplemental readings are the final authority in all factual matters, unless errors have been specifically identified there. You are ultimately responsible for obtaining accurate information when preparing for your exam.

Sigma-delta (Σ-Δ) modulation

- modification of DM: move integrator in RX to position between subtractor and comparator in DM TX, and remove integrator in the feedback loop
- if DM differentiates $m(t)$ in the TX and then integrates $\dot{m}(t)$ in the RX, Σ-Δ modulation reverses the order (integration, then differentiation)
- advantages of DM
    - channel noise does not accumulate in RX's demodulator
    - integrators emphasize low-frequency components (because of multiplication by $1/j\omega$), which are important in speech
    - integrator smooths signal before encoding; thus, slope overload is less likely
    - low-pass nature of integrator reduces variation between samples; thus, smaller encoding error
    - demodulator (RX) is simplified

Transmission of PCM signal over RF channels (digital carrier systems)

- M-ary signaling is possible, where M = no. of possible symbol types sent and $M = 2^n$, where n = no. of bits sent simultaneously
- amplitude-shift keying (ASK)
  - simplest is on-off keying (OOK), which is binary ASK
  - in M-ary ASK, M = no. of amplitude levels sent
- frequency-shift keying (FSK)
  - binary FSK: 1 represented by one frequency; 0 represented by a 2nd frequency
  - in M-ary FSK, M = no. of frequencies used
  - demodulation requires a bank of filters tuned to individual frequencies
  - orthogonal signaling:

    frequencies equally spaced: $f_m = f_1 + (m-1)\delta f$ (δf used b/c Δf is freq. deviation)

    orthogonality: $I = \int_0^{T_b} \left[A_m \cos(2\pi f_m t)\right]\left[A_n \cos(2\pi f_n t)\right] dt = 0 \quad$ if $m \neq n$

    $$I = \frac{A_m A_n T_b}{2}\left\{\frac{\sin\left[2\pi(f_m + f_n)T_b\right]}{2\pi(f_m + f_n)T_b} + \frac{\sin\left[2\pi(f_m - f_n)T_b\right]}{2\pi(f_m - f_n)T_b}\right\}$$

    First term is much smaller than second term, so

    $$I \approx \frac{A_m A_n T_b}{2}\frac{\sin\left[2\pi(m-n)\delta f T_b\right]}{2\pi(m-n)\delta f T_b}$$

    $$\rightarrow \quad \sin\left[2\pi(m-n)\delta f T_b\right] = 0 \quad \text{for } m \neq n$$

    $$\rightarrow \quad 2\pi\delta f T_b = k\pi, \quad k = 1, 2, 3, \ldots \quad \rightarrow \quad \delta f = \frac{1}{2T_b} \text{ is smallest freq. interval}$$

    This is the optimum spacing to make M FSK symbols sufficiently distinct without using too much bandwidth (i.e., needlessly large spacing).
- phase-shift keying (PSK)
  - binary PSK: 1 represented by one phase (e.g., 0°); 0 represented by a 2nd phase (e.g., 180°)
  - in M-ary PSK, M = no. of phase shifts used
  - $M = 2$ corresponds to BPSK (binary PSK); allows one bit to be sent per pulse
  - $M = 4$ corresponds to QPSK (quadrature PSK); allows two bits to be sent at a time
- M-ary quadrature amplitude modulation (M-ary QAM, or M-QAM)
  - can be used to send combinations of k bits at a time, where $M = 2^k$
  - note that k might or might not equal n (no. of bits used to encode PCM samples)
  - 64-QAM and 256-QAM used in IEEE 802.11 "Wi-Fi" wireless networking
  - signal representation:

    $$\varphi_{M-QAM}(t) = a_i p(t)\cos\omega_c t + b_i p(t)\sin\omega_c t = r_i p(t)\cos(\omega_c t - \theta_i), \qquad i = 1, 2, \ldots, M$$

    where $r_i = \sqrt{a_i^2 + b_i^2}$ and $\theta_i = \tan^{-1}\left(\frac{b_i}{a_i}\right)$

    and where a_i and b_i are quantized amplitudes that in combination represent M different bit combinations (usually a power of two), and $p(t)$ = time-domain baseband pulse (with peak amplitude of 1); simplest choice for $p(t)$ is a rectangular pulse, but pulse shaping is usually used to minimize bandwidth
  - ASK and PSK are special cases of M-QAM (ASK is amplitude only; PSK is phase only)

- *constellation diagram*: plot of all possible a_i, b_i amplitude combinations on rectangular coordinate system with a_i plotted along horizontal axis and b_i along vertical axis
    - square constellations are widely used; in this case, M is a power of 4, and points are evenly spaced along a square grid
    - circular constellations also widely used (corresponds to M-PSK); in this case, r_i = constant for all i, and points in constellation are evenly spaced around a circle
    - time-average power of each pulse is given by $r_i^2/2R$, where r_i is the magnitude defined above (assumed to be a *voltage*), and R is the equiv. resistance across which pulse voltage is measured; if r_i is the magnitude of a *current*, then time-average power of each pulse is given by $r_i^2R/2$

Line codes for baseband encoding of digital signals

- on-off keying (RZ)
- polar (RZ)
- bipolar (RZ)
- on/off (NRZ)
- polar (NRZ)
- advantages & disadvantages of various codes
- RZ (return-to-zero) vs. NRZ (nonreturn-to-zero)
- many variations of the above to address specific challenges

Power spectral density (PSD) of line codes

- model the PSD $S_y(f)$ as the product of the PSD of the impulse train $S_x(f)$ and the PSD $|P(f)|^2$ of the pulse shape, where $P(f)$ is the Fourier transform of the pulse; i.e.,

  $$S_y(f) = |P(f)|^2 S_x(f)$$

- Reason: It is impossible to find the Fourier transform of a random impulse train, but the PSD of a random pulse train is the Fourier transform of the autocorrelation $R_x(\tau)$.
- Expression for the PSD becomes

  $$S_y(f) = \frac{|P(f)|^2}{T_b} \sum_{n=-\infty}^{\infty} R_n e^{-j2\pi fnT_b} = \frac{|P(f)|^2}{T_b}\left[R_0 + 2\sum_{n=-\infty}^{\infty} R_n \cos(2\pi fnT_b)\right],$$

  where

  $$R_n = \lim_{N\to\infty} \frac{1}{N}\sum_{k=1}^{N} a_k a_{k+1} \quad \text{and} \quad a_k = \text{amplitude of } k\text{th pulse.}$$

  The second form for $S_y(f)$ given above results because the autocorrelation is an even function, so $R_{-n} = R_n$.
- PSD for polar signaling

  $$S_y(f) = \frac{|P(f)|^2}{T_b}$$

- PSD for on-off keying (OOK)

  $$S_y(f) = \frac{|P(f)|^2}{4T_b}\left[1 + \frac{1}{T_b}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right)\right]$$

- PSD for bipolar signaling

  $$S_y(f) = \frac{|P(f)|^2}{T_b}\sin^2(\pi f T_b)$$

Pulse shaping

- ISI = inter-symbol interference (See Subsec. 6.3.1 of Lathi & Ding, 6th ed.)
- pulse amplitude decision-making: most systems do not need to examine the amplitude of the pulse over the full pulse interval but instead make the amplitude decision at a specific point in time during the pulse interval
- Nyquist's first criterion for zero ISI; assumes that the pulse amplitude decision is made in the middle of the pulse interval; two equivalent tests:

  $$p(t) = \begin{cases} 1, & t = 0 \\ 0, & t = \pm nT_b \end{cases} \quad \text{and} \quad \sum_{n=-\infty}^{\infty} P(f - nR_b) = \text{constant} \text{ (i.e., not a function of } f\text{)},$$

  where $T_b = 1/R_b$ and n is any nonzero integer
- minimum bandwidth pulse for Nyquist's first criterion (impractical; decays as $1/t$):

  $$p(t) = \text{sinc}(\pi R_b t) = \text{sinc}\left(\frac{\pi t}{T_b}\right) \quad \text{and} \quad P(f) = \frac{1}{R_b}\prod\left(\frac{f}{R_b}\right),$$

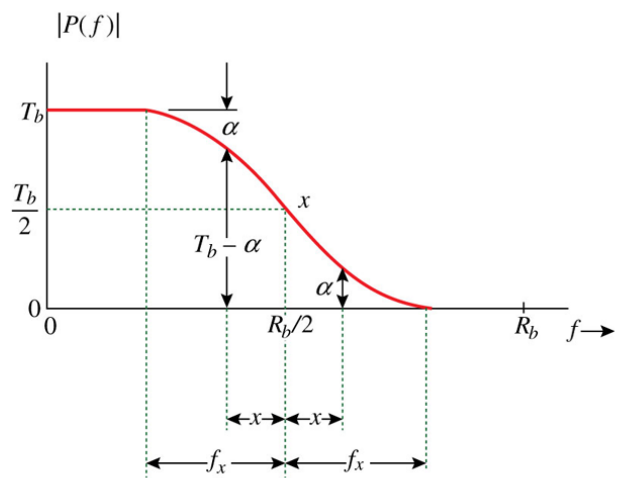
  where $\prod(x)$ is the unit rectangular function that equals 1 for $-0.5 \le x \le 0.5$ and zero outside that range

Practical pulse that satisfies Nyquist's first criterion approximately

- Fourier transform:

  $$P(f) = \begin{cases} 1, & |f| < \frac{R_b}{2} - f_x \\ \frac{1}{2}\left\{1 - \sin\left[\pi\left(\frac{f - R_b/2}{2f_x}\right)\right]\right\}, & \left|f - \frac{R_b}{2}\right| < f_x \\ 0, & |f| > \frac{R_b}{2} + f_x \end{cases}$$

  where f_x is the bandwidth in excess of the minimum bandwidth $R_b/2$
- plot of Fourier transform (Fig. 6.13 of Lathi & Ding, 6th ed.):



- roll-off factor (fraction of frequency range from 0 to $0.5R_b$ over which frequency response rolls off):

  $$r = \frac{f_x}{0.5R_b} = 2f_x T_b, \quad 0 \le r \le 1$$

- prototype freq. response (i.e., the $r = 0$ case) is the case for which there is a sharp cut-off at $f = 0.5R_b$)
- bandwidth of rolled-off pulse:

$$B_T = \frac{(1+r)R_b}{2}$$

- special case with $r = 1$ (raised-cosine pulse):

freq. domain: $P(f) = \frac{1}{2}\left[1 + \cos(\pi f T_b)\right]\Pi\left(\frac{f}{2R_b}\right) = \cos^2\left(\frac{\pi f T_b}{2}\right)\Pi\left(\frac{f}{2R_b}\right)$

time domain: $p(t) = R_b \frac{\cos(\pi R_b t)}{1 - 4R_b^2 t^2}\operatorname{sinc}(\pi R_b t)$ decays as $1/t^3$

Channel effects and equalization

- multipath effects: reflections from objects in the environment that result in delayed copies of transmitted signal arriving at the receiver; reflections are delayed in time and usually attenuated in amplitude
- delay spread
  - widest extent of time delays seen in multipath signals (the range of time delays due to reflections with different path lengths)
  - double, triple, etc. reflections cause even longer delays
  - only reflected signals with significant strength (usually above the ambient noise level) contribute to delay spread
  - key physical relationship

    $$\frac{\Delta R}{\Delta t} = c \quad \rightarrow \quad \Delta t = \frac{\Delta R}{c}, \qquad \text{where } c = 3.0 \times 10^8 \text{ m/s}$$
- fading: attenuation of signals due to multipath, weather, propagation through lossy materials, shadowing, etc.
- equalization:
  - signal processing applied in the receiver meant to offset multipath effects and fading
  - requires knowledge of the propagation channel; channel characteristics are sometimes obtained using known pilot signals sent by transmitter
  - usually implemented using a tapped delay line (FIR filter)
  - many algorithms available to determine coefficients in delay line

Orthogonal frequency division multiplexing (OFDM) – not covered on final exam

<u>Relevant course material:</u>

Homework: #6
Mini-Projects: [none]
Reading: Assignments from Nov. 7 through Dec. 8

This exam will focus primarily on the course outcomes listed below and related topics.

4. Demonstrate how pulse code modulation systems encode analog signals into digital form.
5. Evaluate the basic performance metrics of an M-ary quadrature amplitude modulation (QAM) system.

The course outcomes are listed on the Course Policies and Information sheet, which was distributed at the beginning of the semester and is available on the Syllabus and Policies page at the course web site. The outcomes are also listed on the Course Description page. Note, however, that some topics not directly related to the course outcomes could be covered on the exam as well.